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MAKING SENSE OF MATHEMATICS:

THE *CERTITUDINE MATHEMATICARUM* DEBATE AND

ITS RELATIONSHIP TO PLATO AND ARISTOTLE

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MAKING SENSE OF MATHEMATICS:
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AND ITS RELATIONSHIP TO PLATO AND ARISTOTLE

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To Becky

Thank you for your support and encouragement

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INTRODUCTION

The status of mathematics was fluctuating in sixteenth-century Italy. This was the century in which the humanist revival of mathematics was challenging the traditional Aristotelian understanding. Consequently, it was also a time period in which mathematicians increasingly applied their methods to the topics traditionally treated by Aristotelian natural philosophers. The effect of this was the combination of quantitative methods to a domain that was traditionally qualitative in method.[1] Naturally, this raised all sorts of questions about the proper relationship between the methods of mathematics and that of natural philosophy. One particular one was the debate over the certitude of mathematics.

This debate began in 1547 with the publication of Alessandro Piccolomini's *Commentarium de certitudine mathematicarum disciplinarum.*[2] The main point that Piccolomini made was that mathematics could not fit the logical form of a *demonstratio potissima*, which he understood as providing the most certainty. The implication of this was that mathematics, understood as arithmetic and geometry, could not be used to arrive at the causes of a phenomenon. Instead, as he states, mathematics was "instrumental" to other mixed sciences.[3] Consequently, while the methods of

[1] Peter Dear, "Mixed Mathematics," in *Wrestling with Nature: From Omens to Science*, edited by Peter Harrison, Ronald Numbers, and Michael Shank (Chicago: University of Chicago Press, 2011), p. 149-151.

[2] Alessandro Piccolomini, *Alexandri Piccolominei in mechanicas quaestiones Aristotelis, paraphrasis…Eiusdem commentarium de certitudine mathematicarum disciplinarum* (Roma: Antonio Baldo, 1547).

[3] Piccolomini, *In mechanicas*, p. IIv-IIIr. The specific mixed sciences which Piccolomini lists are Perspectives, Astronomy, Music, Mechanics, Stereometry, and Cosmography.

mathematics were such that mathematics could be easily understood, its application and usefulness was much more limited. In 1560 Francesco Barozzi forcefully opposed Piccolomini in his *Opusculum in quo una ratio, et duae quaestiones: altera de certitudine, et altera de medietate mathematicarum continentur.*[4] This work, published in the year that Barozzi began teaching mathematics at the University of Padua, countered Piccolomini's claim by arguing that mathematics provided the greatest amount of certainty and could be considered a *demonstratio potissima*. Moreover, the way in which Barozzi framed Piccolomini's issue in this work clearly indicates that a significant aspect in this debate was the way in which one was able to appeal to texts in order to legitimize an understanding of the nature of mathematics.

Although the debate itself circulated among the University of Padua in the work of other mathematicians, such as Peter Catena, it is common in the historiography to analyze the way in which the Jesuits appropriated this debate.[5] This was most clearly seen in the work of Benito Pereira, a philosopher and logician, and Christoph Clavius, a mathematician. Both taught at the Collegio Romano at the same time period and both engaged the frameworks of Piccolomini and Barozzi. In 1576 Pereira published *De Communibus omnium rerum naturalium principiis et affectionibus* in which he argued

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[4] Francesco Barozzi, *Opusculum, in quo una Oratio, & duae Quaestiones: altera de certitudine, & altera de medietate Mathematicarum continentur* (Patavii: E.G.P., 1560).

[5] For instance, see Rivka Feldhay, "The use and abuse of mathematical entities," in *The Cambridge Companion to Galileo*, edited by Peter Machamer (Cambridge: Cambridge University Press, 1998), p. 80-145; Chikara Sasaki, *Descartes's Mathematical Thought* (Dordrecht: Kluwer Academic Publishers, 2003), p. 45-63; Peter Dear, *Discipline & Experience: The Mathematical Way in the Scientific Revolution* (Chicago: University of Chicago Press, 1995), p. 32-62. Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Oxford: Oxford University Press, 1996), p. 8-19.

the case that mathematics should not properly be considered a science.[6] Though the work itself was not written specifically about mathematics, it does seem likely that his comments about mathematics are specifically directed toward that of Christoph Clavius. In 1574 Clavius had published his own *Commentary on Euclid*, which contained a Prologue in which he demonstrated the widespread utility and application of mathematics.[7]

The historiography of this debate attests to the fact that most are interested in it for its connection to Jesuit mathematics. It is well known that the Jesuit system of mathematics became preeminent during this time period, and so an analysis of Pereira and Clavius within this debate does provide an insight into the organization of a mathematics curriculum. All of this becomes particularly interesting because it was during this time period that the Jesuits were developing their official educational program, the *Ratio Studiorum*. For Rivka Feldhay the debate over the certainty of mathematics helps to explain the way in which the Jesuits institutionalized mathematics. Moreover, as she contends, since Galileo's science developed out of this program, then one cannot understand the way in which Galileo understood the nature of mathematics without understanding this debate.[8] Chikara Sasaki argues a similar point to that of Feldhay. However, more so than Feldhay, Sasaki is interested in demonstrating the preeminence of Christoph Clavius's role in shaping seventeenth-century mathematics,

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[6] Pereira, *De communibus omnium rerum naturalium principiis and affectionibus, libri quindecim* (Roma: Franciscum Zanettum & Bartholomaeum Tosium Socios, 1576).

[7] Christoph Clavius, *Euclidis Elementorum Libri XV* (Roma: Vincentium Accoltum, 1574).

[8] Feldhay, "The use and abuse of mathematical entities," in *The Cambridge Companion to Galileo*, p. 80-82.

particularly as it helped to shape Descartes's own theory of mathematics.[9] Peter Dear briefly mentions the controversy, and even then only mentions it as an example of a conflict between an Aristotelian system, which he defines by Aristotle's four causes, and a mathematical system. While such a description does address the nature of the debate, Dear's assessment of Clavius is that he essentially presented his position solely based on the authority of Plato.[10] Such a framework, however, fails to appreciate the nuances and particularities of the Jesuit involvement in the wider debate.[11]

However, not all historians address the debate within the context of the Jesuits. Three historians who discuss the controversy beyond the Jesuits are Nicholas Jardine, Paola Mancosu, and Mario Biagioli. Jardine's emphasis is on the logical and epistemological issues at Padua behind the debate. In his opinion there were similar methodological questions being raised in the fields of natural philosophy, medicine, and mathematics, all of which focused on applying logical methods to the Aristotelian corpus.[12] It is quite clear in his presentation that he is predominately influenced by the historiography stemming from John Herman Randall, who understood Padua predominately in the context of logical debates.[13] Paolo Mancosu, who has a widespread interest in the development of the foundation of mathematics, argues that

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[9] Sasaki, *Descartes's Mathematical Thought*, p. 61. According to Sasaki, Clavius was able to achieve a "fresh intellectual atmosphere."

[10] Dear, *Discipline & Experience*, p. 36-37.

[11] Perhaps the most striking aspect of Dear's description of Clavius is his failure to connect Clavius to Francesco Barozzi, a connection that later will be made clear.

[12] Nicholas Jardine, "Problems of Knowledge and Action: Epistemology of the Sciences," in *The Cambridge History of Renaissance Philosophy*, edited by Charles Schmitt, Quentin Skinner, Eckhard Kessler, and Jill Kraye (Cambridge: Cambridge University Press 1988), p. 685.

[13] As will be mentioned later, John Herman Randall significantly defined the historiography surrounding Padua. John Herman Randall, *The School of Padua and the Emergence of Modern Science* (Padova: Editrice Antenore, 1961).

many of the seventeenth century debates over the nature of mathematics are connected to this sixteenth-century debate.[14] Mario Biagioli claims that the debate over the certitude of mathematics that occurred in the second half of the sixteenth century is indicative of the "status-related tensions" between mathematicians and natural philosophers.[15] Biagioli, though, only briefly touches on the debate, rather using it to help make his larger claim that the sixteenth century was a time period in which mathematicians were increasingly being given social authority in positions previously restricted only for natural philosophers.

The historiography, then, is divided as to the significance of the debate over the certitude of mathematics. Most recognize it as influential for later seventeenth century developments, whether in the Jesuit Order or not. A smaller segment, most clearly seen in Mancosu and Biagioli, understand the debate in its particular sixteenth-century context. What is noticeably absent from the historiography is what this debate may suggest about the discourse on mathematics. For instance, how did individuals explain the nature of mathematics? What textual appeals were made? Both Biagioli and Dear come the closest to such an analysis, but oversimplify the nature of the debate among those involved. This study will work to analyze the debate, particularly with respect to the structures of argumentation involved, in order to elucidate aspects of the nature of mathematical discourse in late sixteenth-century Italy.

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[14] Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, p. 8-10. Mancosu specifically has in mind Blancanus, Smiglecius, Barrow, Gassendi, Hobbes, and Wallis.

[15] Mario Biagioli, "The Social Status of Italian Mathematicians, 1450-1600," *History of Science* 27 (1989): p. 54.

THE PLACE OF MATHEMATICS IN RENAISSANCE ITALY

The Renaissance was a period of substantial mathematical development among the universities and academies. Paul Grendler notes three in his study of Italian Universities during the Renaissance. First, it was during this time period that algebra and geometry grew in importance alongside Arithmetic. Second, astronomy slowly began to supplant astrology as dominant. And third, the category of "mathematician" slowly increased in importance alongside that of the natural philosopher.[16] While the debate over the certitude of mathematics was related to these developments, it was much more closely connected to concerns over the classification of the sciences and the application of logical methods. Due to the Renaissance recovery of the ancient Greek texts, especially the commentators on Aristotle, the significance and interpretation of Aristotle was being questioned.[17] It was within such a context that the debate over the certitude of mathematics occurred. All of those involved were trying to understand the relationship of the classification of the sciences in light of recently recovered Greek mathematical texts. Consequently, one cannot understand the epistemological interests that Piccolomini, Barozzi, Pereria, and Clavius had until first understanding the basic organization of the sciences from which they operated.

As James Weisheipl explains, the purpose of classifying the sciences was to determine the relationship between various disciplines in order to determine what was true *scientia*, or "knowledge." Its basic framework was established by Plato, but was

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[16] Grendler, *The Universities of the Italian Renaissance* (Baltimore: Johns Hopkins University Press, 2002), p. 408-410

[17] Charles B. Schmitt, *Aristotle and the Renaissance* (Cambridge, MA: Harvard University Press, 1983), p. 10-33.

later commented on and modified by Aristotle and subsequent philosophers. According to Plato the certainty of knowledge was a direct correlate to its abstractness; the more abstract the knowledge the more certain it was.[18] This intellectual framework, based on both the *Republic* and the *Timaeus*, stated that only the immutable was real, and consequently true knowledge was attained as one contemplated the abstract in search of the eternal. To help organize the various types of knowledge in relationship to his intellectual schema, Plato codified the speculative science into dialectics, mathematics, and natural philosophy. In his schema natural philosophy was the lowest form of knowledge, mathematics the middle, and dialectics, a process of questioning in search of general comprehensive principles, was the highest form of knowledge. For Plato, the mathematical disciplines, comprised of geometry, astronomy, arithmetic, and harmonics, were more real than natural philosophy due to their mathematical nature. Because of this mathematics was the avenue through which one was able to enter dialectics, the highest science, and the one that would enable one to understand the immutable beings.[19]

For Aristotle, though, it was different. He rejected Plato's idea that the speculative sciences provided an ordering of knowledge or of being. For him, the middle position of mathematics indicated its relationship to both the natural sciences and metaphysics. For Aristotle, physics was the truest science, as it enabled one to comprehend the true nature of a being. For him the purpose of mathematics was that it was the most clearly accessible to the senses and thus most easily comprehended.

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[18] Cf. *Republic* 6 509D-511E.

[19] James Weisheipl, "Nature, Scope, and Classification of the Sciences," in *Science in the Middle Ages*, edited by David Lindberg (Chicago: University of Chicago Press, 1978), p. 461-466.

Consequently, it provided a beneficial introduction to the natural sciences and metaphysics, the so-called "sciences of being," of physics, metaphysics, and ethics. As compared with Plato, Aristotle's system of the sciences was much more complex, organized instead by practicality, with three main sections, rather than a hierarchy of being. The three parts to his system were the arts, the practical sciences of moral philosophy, and the speculative sciences, which included physics, mathematics, and metaphysics.[20]

Of the two, Aristotle provided the clearest explanation of the nature of scientific knowledge, most clearly explained in his *Posterior Analytics*. According to Aristotle's system, knowledge was established through syllogisms, which were made up of three parts. All syllogisms were composed of a subject and a predicate, with a middle term that expressed the causal relationship between the subject and the predicate. And, within syllogisms there were two main types. The first, and the most authoritative, was the "demonstration of the reasoned fact," which later became known as the *demonstratio propter quid* by those in the middle ages. The second, "the demonstration of the fact," later known as the *demonstratio quia*, was the type of demonstration that was most readily available. The *demonstratio quia* provided the "proper, immediate, and commensurate" cause of an observed effect or fact. The form of the *demonstratio quia* was often that of facts, conjectures, hypotheses, and opinions of individuals.[21] One significant difference between the two was that the *propter quid* explained effects through their causes whereas the *demonstratio quia* proceeded from the effects to the causes.

[20] Ibid., p. 466-467.
[21] Ibid., p. 467-468.

Aristotle makes clear in *Posterior Analytics* I.13 that the type of knowledge derived by each of these logical demonstrations differed. The example provided is a proof for the proximity of the planets to the earth. According to Aristotle one could argue as follows: 1) The planets do not twinkle. 2) What does not twinkle is near the earth. 3) Therefore, the planets are near the earth. Such a construction, however, would only provide knowledge of the effect, *demonstratio quia*, since the non-twinkling of the planets was not the cause of their proximity. To turn this into a *demonstratio propter quid*, one could argue as follows: 1) The planets are near the earth. 2) What is near the earth does not twinkle. 3) Therefore the planets do not twinkle. In constructing the example as this, it is noticeable that the nearness of the planets was the cause that expressed the non-twinkling of the planets. As Paolo Mancosu indicates with regard to this example, the essential difference between a *quia* and a *propter quid* that Aristotle was indicating, was whether or not the middle term, the aspect of a syllogism which expressed the relationship between the subject and its predicate, could express the proximate cause.[22]

An essential part of Aristotle's framework of *demonstratio propter quid* and *demonstratio quia* was the fact that a scientific demonstration could not transfer from one genus to another.[23] Crossing types of demonstrations would only achieve accidental understanding, whereas a non-accidental demonstration occurred when the middle term of a syllogism was of the same type as the subject and its predicate. An exception to this, as Steven Livesey notes in his study of the fourteenth century scholastic John of

[22] Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, p. 12.

[23] Cf. *Posterior Analytics* I.7 75

Reading, is the connection between the mathematical sciences and other disciplines, such as in the subalternation of the sciences.[24] In Aristotle's *Posterior Analytics* I.9 Aristotle notes the way in which sciences of two different types could be connected. The specific situation envisioned is the relationship of one science subalternated to another science, such as mechanics to geometry.[25] According to Aristotle the inferior science would oftentimes understand by way of *demonstratio quia* whereas the superior science would understand by way of *propter quid*. In the example of the mechanical arts and geometry, someone practicing the mechanical arts might understand certain effects in certain situations, thereby through *demonstratio quia*, whereas the geometer would actually be able to explain the causes, thus *demonstratio propter quid*.[26]

Steven Livesey notes some important observations from all of this. According to Aristotle, the similarity of the subject matter between mechanics and geometry was not enough to demonstrate a similarity in their type of demonstration. Yet, for Aristotle, the two could still be successfully combined together. Aristotle's solution to this was to point out that the inferior science, in this case mechanics, depended on geometry for its principles and its proofs.[27] The implication of this was that two sciences of two different types could successfully be considered as connected together if their process of investigation could be understood as interrelated together.

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[24] Steven Livesey, *Theology and Science in the Fourteenth Century: Three Questions on the Unity and Subalternation of the Sciences from John of Reading's Commentary on the Sentences* (Leiden: Brill, 1989), p. 23.

[25] *Posterior Analytics* I.9 76. Other examples include arithmetic to music, optics to geometry, star-gazing to astronomy.

[26] *Posterior Analytics*, I.9 76.

[27] Livesey, *Theology and Science in the Fourteenth Century*, The specific term is *modus considerandi*. p. 23-24. Cf. *Posterior Analytics* I.9 76 and I.12 77.

Within Aristotle's classification of the sciences mathematics was considered to be a superior science. But, it is also evident that such an understanding was connected to Aristotle's notion of the subalteration of the sciences. Pure mathematics, namely arithmetic and geometry, were considered tools connected to the study of other subject matters. This was, however, in sharp contrast to Plato, for whom pure mathematics held a self-evident position, due to its connection with the ideal forms. When comparing the justification of mathematics in the work of both Plato and Aristotle, it is noticeable that the ontological justification of mathematics differs. In Plato's system, mathematics is presumed of benefit based on the assumption of the ideal forms, whereas in Aristotle's mathematics was presumed less essential due to his emphasis on the nature of an object.

The followers of Plato and Aristotle organized their systems of knowledge in varying ways, the most important of which was Boethius in his *De Trinitate,* who clearly outlined the organization of the sciences.[28] His work provided a significant presentation of the liberal arts system, used throughout the medieval period, largely based on the three speculative sciences, *scientia naturalis*, *scientia mathematica*, and *scientia theologica*. Each of these had its own method associated with the nature of study. The method of *scientia naturalis*, which considered objects connected to both form and motion, was the process of reason. The method of *scientia mathematica* was disciplinary, as it helped to explain a variety of disciplines.[29] And the method of *scientia theologica* was a process of intellectual contemplation in search of the pure being.[30]

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[28] Weisheipl, *Nature and Motion in the Middle Ages*, p. 209.

[29] Ibid., p. 210. Early on the disciplines were the Quadrivium: astronomy, geometry, music, and arithmetic. But other disciplines were added to them, such as optics, stereography, stenography, and cartography.

[30] Ibid., p. 209-210.

However, to appreciate the system that Boethius had devised, and the way in which it was later interpreted, it is important to realize that the system itself was a fusion of both Aristotelian and Platonic elements. As James Weisheipl points out, the status of physics as capable of *scientia* was something that developed from Aristotle, and that of mathematics from Plato and Pythagoras.[31] Although much of this would change throughout the middle ages, and various differences would develop, the basic tripartite organization continued up through the Renaissance.[32] Consequently, for Piccolomini, Barozzi, Clavius, and Pereira to debate the status of mathematics was simultaneously to engage the classification of the sciences.[33]

Historians have noted that the classification of the sciences, and their relationship to Aristotle's logic and *Posterior Analytics*, was a common topic throughout the Renaissance, especially at Padua, where both Piccolomini and Barozzi were located.[34] This aspect was given particular attention in the work of John Herman Randall, who argued that the scientific method associated with Galileo and the seventeenth century were logical developments from the ancient and medieval period that had come to fruition in the Renaissance, especially in the work of Giacomo Zabarella and the School of Padua.[35] One particular aspect that Randall pointed

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[31] Ibid., p. 210.

[32] Ibid., p. 211-237.

[33] Note that both Piccolomini and Pereira begin their treatises invoking the classification of the sciences. Cf. Piccolomini, *In Mechanica*, p. Ir-IIv; Pereria, *De principiis*, p. 3-5.

[34] Heikki Mikkeli, *An Aristotelian Response to Renaissance Humanism: Jacopo Zabarella on the Nature of the Arts and Sciences*, p. 35-36; Roger Ariew, "Christopher Clavius and the Classification of the Sciences," 83, no. 2 (1990): p. 293.

[35] Randall, *The School of Padua and the Emergence of Modern Science*, p.25-27. Since the time of Randall historians have noted the oversimplification provided by Randall. See also Mikkeli, *An Aristotelian Response to Renaissance Humanism: Jacopo*

attention toward was the idea of the "regressus." The regressus was a logical construction intended to allow one to determine the absolute cause of an event only having the proximate causes.[36] According to Nicholas Jardine, a significant Aristotelian principle upon which the regressus developed was Aristotle's distinction between "absolute" and "accidental" knowledge. Whereas the accidental knowledge was better grasped by one's senses, the absolute knowledge was better grasped according to an object's nature.[37] By employing the regressus, and its method of exchange between observed effect and demonstrated knowledge, one could possibly arrive at the knowledge of the cause.[38] And, it was believed that through such a process one could achieve the *demonstratio potissima*, the "absolute demonstration" of the effect, and consequently absolute knowledge. Piccolomini refers on occasion to the concept of the "regressus," indicating that involved in his critique of mathematics were similar logical concerns as those associated with the regressus.[39]

Historically it should be noted that the *demonstratio potissima* developed from the Proemium of Averroes's commentary on Aristotle's *Physics*. Whereas Aristotle

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Zabarella on the Nature of the Arts and Sciences, p. 35-36; Roger Ariew, "Christopher Clavius and the Classification of the Sciences," *Synthese* 83, no. 2 (1990): p. 293; Charles Schmitt, p. 107.

[36] Randall, *The School of Padua and the Emergence of Modern Science*, p. 25. For an overview of the *regressus*, see Jardine, "Problems of Knowledge and Action: Epistemology of the Sciences," in *The Cambridge History of Renaissance Philosophy*, p. 686-688.

[37] Cf. *Posterior Analytics* I.2; *Prior Analytics* II.23; *Physics* I.I

[38] Jardine, "Problems of Knowledge and Action: Epistemology of the Sciences," in *The Cambridge History of Renaissance Philosophy*, p. 687. Jardine lays out a general framework. 1) Through observation one arrives at 'confused' knowledge of an effect. 2) By induction along with demonstration of the fact, one obtains accidental knowledge of its cause. 3) Through a "noetic" process, one obtains absolute knowledge of the proximate cause. 4) By demonstration of the reasoned fact, absolute knowledge of the effect is obtained.

[39] Piccolomini, *Commentarium*, p. LXXXIIIr-LXXXIIIv.

himself had only mentioned two different types of logical demonstrations, the *demonstratio propter quid* and the *demonstratio quia*, Averroes had added a third which was supposed to be the strongest form of demonstration, the *potissima*. As he explains in his Proemium, the *demonstratio potissima* was a combination of the *demonstratio propter quid* and *demonstratio quia*, in which one is trying to arrive at the proximate cause from the remote cause. It was a proof that could both demonstrate the cause of a phenomenon as well as the existence of it.[40] During the Renaissance, and especially at Padua, Averroes's extended analysis of method was popular among those involved with understanding Aristotle's theories of demonstration.[41] However, for Piccolomini, and the others involved in the debate, the rhetorical significance of the *demonstratio potissima* was that it was considered to be the strongest form of syllogism. Thus, as it relates to mathematics, the implication was that mathematics could not be used to provide the strongest form of demonstration.[42]

But the debate over the certitude of mathematics involved more than Aristotelian logic. It was also deeply influenced by humanist interests, an aspect that was particularly important for the development of mathematics during the sixteenth

[40] Daniele Cozzoli, "Alessandro Piccolomini and the Certitude of Mathematics," *History and Philosophy of Logic* 28, no. 2 (2007): p. 158. See also Jardine, "Problems of Knowledge and Action: Epistemology of the Sciences," in *The Cambridge History of Renaissance Philosophy*, p. 688.

[41] Charles Burnett and Andrew Mendelsohn "Aristotle and Averroes on Method in the Middle Ages and Renaissance: The "Oxford Gloss" to the *Physics* and Pietro D'Affeltro's *Expositio Proem Averroys*," in *Method and Order in Renaissance Philosophy of Nature: The Aristotle Commentary Tradition*, edited by Daniel A. Di Liscia, Eckhard Kessler, and Charlotte Methuen (Aldershot: Ashgate, 1997), p. 53-111.

[42] Though it likely develops from his humanist sensibilities, Piccolomini does focus some of his attention in demonstrating that the *potissima* was not merely a development of Averroes, but was somewhat related to Aristotle, especially as explained by two his commentators, Simplicius and Themistius. See Piccolomini, *Commentarium*, p. LXXXIIIr-LXXXIIIv

century. Historically, this aspect has been clearly shown by Paul Rose in his *The Italian Renaissance of Mathematics*.[43] According to Rose during the fifteenth and sixteenth centuries, the humanists, most notably in Florence, Rome, and Venice, were active in the recovery of the Greek mathematical texts, such as the Aristotelian *Mechanica,* Archimedes, Hero, Pappus, Euclid, Apollonius, Ptolemy, and Theodosius.[44] These humanists, such as Regiomontanus, Francesco Maurolico, and Federico Commandino travelled throughout Europe, recovered manuscripts, edited them, frequently published them, and then oftentimes translated them into Latin. Aside from doing their work out of reverence for the classical sources, Rose notes that many of these humanists were inspired by the techniques in the manuscripts, and would oftentimes work to reproduce them.[45] Of those involved in the debate over the certitude of mathematics, both Piccolomini and Barozzi most clearly fit the pattern of Renaissance humanists interested in the recovery of Greek sources. Yet, as will be addressed later, the difference in their humanist aspirations contributed to the difference in their perspective.

Before addressing the particularities of the texts of Piccolomini, Barozzi, Pereira, and Clavius, the importance of two texts should be briefly mentioned. The principle mathematical text about which these theorists were debating was Euclid's *Elements*. Euclid had been important since the twelfth century revival of mathematical

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[43] Paul Rose, *The Italian Renaissance of Mathematics: Studies on Humanists and Mathematicians from Petrarch to Galileo* (Genève: Librairie Droz, 1975).

[44] Paul Rose, "Humanist Culture and Renaissance Mathematics," *Studies in the Renaissance* 20 (1973): p. 2; 104-105.

[45] Rose, *The Italian Renaissance of Mathematics*, p. 164-168; 209-214; 292-294. According to Rose, the most sought after texts during this period were Archimedes, Euclid, Appolonius, and Diophantus.

texts. His emphasis on geometry was of particular importance for the practices of astronomy and astrology.[46] Euclid was also important for the Renaissance, providing a much more robust philosophy for mathematics than that of Archimedes. Prior to the Renaissance the text was only known in fragmentary form in Europe. In the twelfth century Gerard of Cremona's translation of the *Elements* into Latin from Arabic, proved significant for the preservation of Euclid's text. However, even though the Europeans had a Latin translation of the *Elements* in the middle ages, there was still considerable disagreement as to the mathematical content of the work itself up through the Renaissance.[47]

In 1482 Erhard Ratdolt produced the first printed Latin text of Euclid's *Elements*, largely based on the medieval version of Euclid by Campanus of Novara. Simon Grynaeus produced the first Greek critical edition of Euclid in 1533. Scholars have noted that Grynaeus's text was particularly important in its presentation, because Grynaeus sought to separate out the authentic aspects of Euclid from the spurious ones.[48] It was within such a context, one in which the restoration of Euclid was an important component, that the debate over the certitude of mathematics occurred.

Another main text that was used was Proclus's *Commentary on the First Book of Euclid*. As will be shown later, this text was assumed among those involved to be an authoritative text, if not the most authoritative text, to define the nature of mathematics.

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[46] Grendler, *The Universities of the Italian Renaissance*, p. 408-409.

[47] Michael S. Mahoney, "Mathematics," in *Science in the Middle Ages*, p. 149-152. For an analysis of the way in which Euclid was read during the medieval period, see John E. Murdoch, "Transmission into Use: The Evidence of Marginalia in the Medieval Euclides Latinus," *Revue d'histoire des science* 56, no. 2 (2003): p. 369-382.

[48] Dale Billingsley, "Authority in Early Editions of Euclid's *Elements*," *Fifteenth Century Studies* 20 (1993): p. 6-7.

Consequently, the particularities of Proclus's text must be addressed before attending to the particularities of the debate itself.

PROCLUS'S COMMENTARY ON EUCLID IN THE SIXTEENTH CENTURY

Historians have recognized the centrality of Proclus's *Commentary on the First Book of Euclid* in all the Renaissance discussions of mathematics.[49] The debate over the certitude of mathematics is included in this. Although there is not agreement as to its precise influence, and no historian to date has written the narrative of its reception during the Renaissance, it is likely that part of the significance of this text is on account of two factors. The first is due to the fact that the two Prologues of this text provide a very detailed framework through which to understand the theory of mathematics, of which Geometry is given a particularly elevated position. The second is due to the fact that Proclus connected a Platonic framework of mathematics with an Aristotelian system of demonstration. Both of these factors proved to be very important in the sixteenth-century controversy over the certitude of mathematics.

Proclus was a fifth century philosopher in the Byzantine Empire, who spent part of his life as the head of the philosophy school in Athens.[50] A prolific author, only one-

[49] Alistair Crombie, *Styles of Scientific Thinking in the European Tradition, Volume I* (London: Duckworth, 1994), p. 285; Ian Stewart, "Mathematics as Philosophy: Barrow and Proclus," *Dionysius* 18 (2000): p. 151-152; Guy Claessens, "Clavius, Proclus, and the Limits of Interpretation: Snapshot-Idealization Versus Projectionism," *History of Science* 47 (2009): p. 319; The most curious historiographical treatment of Proclus is by Robert Westman, who claims that Proclus's work was influential in solidifying the validity of astrology. However, I did not find this in any other texts, and it is not entirely clear what motivated Westman to make such a claim. See Robert Westman, *The Copernican Question: Prognostication, Skepticism, and Celestial Order* (Berkeley: University of California Press, 2011), p. 202.

[50] Lucas Siorvanes, "Proclus' life, works, and education of the soul," in *Interpreting Proclus: From Antiquity to the Renaissance*, edited by Stephen Gersh (Cambridge, UK: Cambridge University Press, 2014), p. 34.

third of his fifty-four works survive from antiquity.[51] Throughout his life Proclus taught on a wide range of topics, such as Aristotle, Plato, Euclid, Ptolemy, and Homer. Proclus's epistemological framework emphasized a progression from the things most easily perceived to things more abstract. Within such a framework mathematics held a particularly beneficial position as it was in a transitory position, between the sensory world and the world of abstraction. Consequently, for Proclus mathematics was the essential mechanism through which an individual could come to understand the world of imaginary forms.[52] Nevertheless, his work also demonstrates his understanding of Aristotle's theories of demonstration, and throughout his text he works to integrate an Aristotelian theory of demonstration amidst a thoroughly Platonic framework for mathematics. Although it has yet to be conclusively proven, some historians have suggested that the influence of his work in the Renaissance was on account of this aspect.[53]

The entirety of Proclus's *Commentary on the First Book of Euclid* did not survive from antiquity. What did survive were two different prologues, Proclus's explanation of the definitions, postulates, and axioms of Euclid's *Elements Book I*, and also Proclus's commentary on the theorems of Euclid's Book I. It is clear from the beginning of Proclus's text that he intended mathematics to be understood in the context

[51] Lucas Siorvanes, "Proclus' life, works, and education of the soul," in *Interpreting Proclus*, p. 34.

[52] Ibid., p. 43. Though not emphasized in his *Commentary on the First Book of Euclid*, in some of his more philosophical works the ideal that he emphasized was that which Plato emphasized in the *Dialogues*.

[53] On this point, see Claessens, "Clavius, Proclus, and the Limits of Interpretation," p. 318-319.

of Platonic philosophy. At the beginning of the first prologue, when Proclus is setting the broad framework for mathematics, he writes

> Mathematical being necessarily belongs neither among the first nor among the last and least simple of the kinds of being, but occupies the middle ground between the artless realities - simple, incomposite, and indivisible - and divisible things characterized by every variety of composition and differentiation.[54]

Moreover, as Proclus explains, mathematical entities are not mere abstractions, and thus derivative of sense objects, but are objects connected with the higher reality, which he refers to as the Nous.[55] When discussing Euclid, Proclus states

> Euclid belonged to the persuasion of Plato and was at home in this philosophy; and this is why he thought the goal of the *Elements* as a whole to be the construction of the so-called Platonic figures.[56]

The second prologue of Proclus's text specifies the importance of Geometry. Since geometry was the mathematics of Euclid's *Elements*, this topic was a natural progression after his general introduction. It is interesting to note that of the two pure mathematical sciences, geometry and arithmetic, arithmetic was considered the highest. As he states, "everything that is expressible and knowable in geometry is determined by arithmetical ratios."[57] Nevertheless, it is clear Proclus's main interest was geometry. He stated geometry "looks around upon the region of genuine being, teaching us through images the special properties of the divine orders and the powers of the intellectual forms, for it contains even the ideas of these beings within its range of vision."[58] He then explained that the reason geometry works is because the universals are in someway

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[54] Proclus, *A Commentary on the First Book of Euclid's Elements*, translated by Glenn Morrow (Princeton: Princeton University Press, 1970), p. 3.
[55] Ibid., p. 13-14.
[56] Ibid., p. 57.
[57] Ibid., p. 39.
[58] Ibid., p. 50.

associated with the various particulars.[59] He explained that the subject matter of geometry is magnitudes, figures and their boundaries, ratios, properties, physical positions, and motions, which make use of "hypotheses" and "first principles" which it receives from the arithmetic.[60]

When Proclus organizes the sciences of mathematics he claims that he is doing it in such a way that would accord with Aristotle. According to Proclus, Aristotle emphasized that the sciences that begin from simpler principles are more accurate than others. Proclus claims this is what he has done in organizing the relationship between arithmetic and geometry, but also other sciences such as music and arithmetic and geometry and mechanics.[61] The significance of this is that it indicates the way in which Proclus developed the theory of mathematics in his Prologue in explicit recognition of the way in which it compared to Aristotle's own classification of the sciences.

Yet, Proclus's understanding of science was not merely one of similarity to that of Aristotle. It was also one of difference. At the beginning of the section in which he discusses the theorems of Euclid's Book I, Proclus addresses the relationship between geometrical proofs and Aristotle's insistence on demonstrative causation. It was Proclus's belief that geometry was able to provide the same knowledge as that of causation, as it was able to answer the question "why" something occurs. However, as he admits, the nature of geometrical proofs means that it will arrive at this in a different way than Aristotle's system. As he states,

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[59] Ibid., p. 40-41. He specifically lists three relationships. "the universal shared in by its particulars, the universal in its particulars, and the universal that supplements the particulars."

[60] Ibid., p.46.

[61] Ibid., p.47-48.

> What is called "proof" we shall find sometimes has the properties of a demonstration in being able to establish what is sought by means of definitions as middle terms, and this is the perfect form of demonstration; but sometimes it attempts to prove by means of signs….Although geometrical propositions always derive their necessity from the matter under investigation, they do not always reach their results through demonstrative methods.[62]

To explain his point he used Euclid's proof of proposition I.32. This proposition states,

> In any triangle, if one of the sides is produced, the exterior angle of the triangle is equal to the two interior and opposite angles, and the interior angles of the triangle are equal to two right angles.[63]

In order to prove this proposition, as Figure 1 demonstrates, it requires the construction of another angle, bisecting the exterior angle. As Proclus explains, however, this proposition cannot be explained as a demonstration based upon the cause, since "it is a triangle even if its side is not extended."[64] By this Proclus appears to be indicating that the proof itself does not operate according to any causal constructions, but instead is dependent on the properties of a triangle itself.

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[62] Ibid., p. 161.

[63] *Euclid's Elements: All Thirteen Books Complete in One Volume*, translated by Thomas L. Heath, edited by Dana Densmore (Santa Fe: Green Lion Press, 2002), p. 24-25.

[64] Proclus, *A Commentary on the First Book of Euclid's Elements*, p. 162.

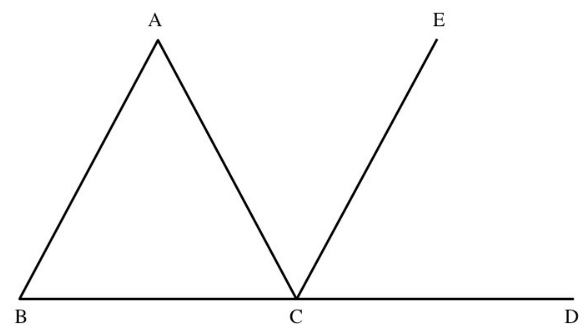

Figure 1[65]

However, as Proclus explains, if an individual was to construct a triangle based upon the drawing of circles and the result was an equilateral triangle, then the triangle could be proven demonstrably. As Proclus explains, "it is the similarity and equality of the circles that causes the equality of the sides of the triangle."[66] The proposition that this refers to in Euclid is I.I. This proposition states, "On a given finite straight line to construct an equilateral triangle."[67] As Figure 2 indicates, the proof itself depends on the construction and qualities of two circles.

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[65] According to Euclid, the proof is as follows:
1) Let CE be drawn through the point C parallel to the straight line AB.
2) Then, since AB is parallel to CE, and AC has fallen upon them, the alternate angles BAC, ACE are equal to one another.
3) Again, since AB is parallel to CE, and the straight line BD has fallen upon them, the exterior angle ECD is equal to the interior and opposite angle ABC.
4) But the angle ACE was also proved equal to the angle BAC; therefore the whole angle ACD is equal to the two interior and opposite angles BAC, ABC.
5) Let the angle ACB be added to each; therefore the angles ACD, ACB are equal to the three angles ABC, BCA, CAB.
6) But, the angles ACD, ACB are equal to two right angles; therefore the angles ABC, BCA, CAB are also equal to two right angles."
Euclid's Elements, p. 24-25. Cf. *Posterior Analytics* I 23.
[66] Ibid., p. 162.
[67] *Euclid's Elements*, p. 3.

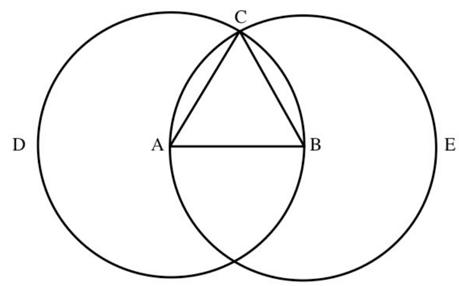

Figure 2[68]

For Proclus neither of these differences in proof affects the status or properties of the triangle. Instead, the significance resides in the nature of the proof. One is causally demonstrated, the other based on added elements. Most importantly, both for Proclus should have the equal status of “proof”. Throughout the debate over the certitude of mathematics Proclus’s analysis of both propositions will be used to indicate the nature of a geometrical demonstration.

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[68] According to Euclid, the proof is as follows:
1) Let AB be the given finite straight line.
2) Thus it is required to construct an equilateral triangle on the straight line AB.
3) With Centre A and distance AB let the circle BCD be described;
4) again, with centre B and distance BA let the circle ACE be described;
5) and from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined.
6) Now, since the point A is the centre of the circle CDB, AC is equal to AB.
7) Again, since the point B is the centre of the circle CAE, BC is equal to BA.
8) But CA was also proved equal to AB; therefore each of the straight lines CA, CB is equal to AB.
9) And things which are equal to the same thing are also equal to one another;
10) therefore CA is also equal to CB.
11) Therefore the three straight lines CA, AB, BC are equal to one another.
12) Therefore the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB.” *Euclid’s Elements*, p. 3.

More than anything else, Proclus's *Commentary on the First Book of Euclid* was a comprehensive introduction to mathematics. Not only did it generally introduce mathematics, but it also specifically addressed geometry. In addition, due to his historical proximity, as coming after both Plato and Aristotle, it is evident that he drew together aspects of both Plato and Aristotle into comparison with each other. While his work should not be recognized as an integration, since Proclus was self-consciously a follower of Plato in his epistemology, it is likely that one of his appeals throughout the Renaissance resulted from not only his comprehensive introduction but also the ability of his text to appeal both to Platonic and Aristotelian epistemologies.[69]

According to Stephen Gersh the *fortuna* of Proclus in the medieval and Renaissance period are in contrast to each other. Although Proclus's works were basically available in Greek from antiquity until the Renaissance, before the Renaissance period, Proclus was largely looked upon either as a religious secularist or a plagiarist.[70] Until the Renaissance period it was largely presumed that Proclus's works were plagiarized from Dionysius the Areopagite, the supposed convert of the Christian Apostle Paul found in the biblical text Acts 17:34.[71] Thus, until the Renaissance period, Proclus's texts, including his commentary on Euclid, were likely not widely known.

However, towards the end of the fifteenth century there was a rediscovery of the value of Proclus's work. Its earliest usage attests to its benefit for mathematics. One of

[69] For an overview of the way in which both Plato and Aristotle were being compared, see Brian Copenhaver and Charles Schmitt, *Renaissance Philosophy* (Oxford: Oxford University Press, 2002), p. 127-142.

[70] Stephen Gersh, "One thousand years of Proclus," in *Interpreting Proclus*, p. 3.

[71] Ibid., p. 6-8. As Gersh points out, although now Dionysius the Areopagite is referred to as "Pseudo-Dionysius the Areopagite" such a conclusion was not discovered until the nineteenth century, at which point Dionysius came to be recognized as a sixth century figure.

the earliest, Giorgio Valla quoted from Proclus's commentary in his *Concerning What to Seek and What to Shun*, naming it as a beneficial text for understanding the theory of mathematics.[72] Zamberti began a Latin translation of Proclus's commentary on Euclid in 1539, although he did not publish it.[73] The first person to print Proclus's commentary on Euclid in Greek was the humanist and philologist Simon Grynaeus in 1533, who appended it to his Greek edition of Euclid, the same edition mentioned above which historians have recognized as a significant critical edition of Euclid.[74] In the preface to the work Grynaeus pointed out that geometry was important not only for studying the sensible world, but also because mathematics was similar to logic, as a resource to organize all the arts.[75]

Robert Goulding adds a further aspect with regard to the influence of Proclus's commentary. According to Goulding Proclus's historical summary, which Proclus includes in his second prologue, provided a valuable historical account of the development of mathematics that appealed to humanist historical sensibilities.[76] According to Proclus mathematics began out of practical need. As he relates, the beginning of mathematical development was with the Egyptians, who developed geometry in order to help them restore boundary lines when the Nile flooded. After the Egyptians the next great advancement was the Phoenicians, who developed number theory in order to help them with trade transactions. Following the Phoenicians, Proclus

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[72] Robert S. Westman, *The Copernican Question*, p. 202.

[73] Rose, "Bartolomeo Zamberti's Funeral Oration for the Humanist Encyclopaidist Giorgio Valla," in *Cultural Aspects of the Italian Renaissance: Essays in Honor of Paul Oskar Kristeller*, p. 299-310.

[74] Westman, *The Copernican Question*, p. 202.

[75] Ibid., p. 202.

[76] Robert Goulding, *Defending Hypatia: Ramus, Savile, and the Renaissance Rediscovery of Mathematical History* (Dordrecht: Springer, 2010), p. 7.

emphasizes the development of mathematics among the Greeks, beginning with Thales, then the Pythagoreans, and then climaxing with the achievement of Plato and the school he had established.[77] For the humanists, who were deeply interested in the historical origins, Proclus's history of mathematics was of immense importance.[78]

Thus, Proclus's *Commentary on Euclid* appealed to a variety of audiences. Its historical accounting appealed to humanist sensibilities, and its integration of Plato and Aristotle together appealed to those who were involved in such debates. In addition, for the reception of Euclid, and the development of mathematical theory more broadly, the theoretical framework contained in the opening prologues proved of immense value. For those involved in the debate over the certitude of mathematics, each of these aspects was quite important. Piccolomini is clear in his text that he is appealing to a Proclean framework of mathematics when he claimed that mathematics did not fit a *demonstratio potissima,* and then a significant part of Barozzi's response was his translation of Proclus's *Commentary* into Latin. Thus, even though it would be historically insufficient to causally claim that Proclus's text inspired the debate, it is certain that his text was significantly involved early on in the progression of the debate. Consequently to participate in the debate was simultaneously to engage the text of Proclus.

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[77] Proclus, *A Commentary on the First Book of Euclid's Elements*, translated by Glenn Morrow, p. 52-56. It is interesting to note in this section the emphasis that Proclus places upon Plato as the central figure. Everyone after Plato, who was a student of Plato or involved in Plato's school, has such a designation specifically attached to their name. Such as, "Amylase of Heracleia, one of Plato's followers, Menaechmus, a student of Eudoxus who also was associated with Plato."

[78] For a background on mathematical history and humanism, see James Steven Byrne, "A Humanist History of Mathematics? Regiomontanus's Padua Oration in Context," *Journal of the History of Ideas* 67, no. 1 (2006): p. 41-61.

THE *CERTITUDINE MATHEMATICARUM* DEBATE: PICCOLOMINI AND BAROZZI

The beginning of the debate over the certitude of mathematics focused on the nature of mathematics itself. For Piccolomini, the abstract nature of mathematics precluded its possibility of being a *demonstratio potissima*, for Barozzi it was the abstract character that legitimized it as a *demonstratio potissima*. Yet throughout their debate it becomes clear that a significant factor involved in the debate was how to legitimate their positions, especially with regard to which textual authorities could be utilized to defend their position.

ALESSANDRO PICCOLOMINI: MATHEMATICS ARE ABSTRACTION

When Alessandro Piccolomini raised the question about the certitude of mathematics, he based his opinion on two essential principles. The first was that logic, as defined by Aristotle in the *Posterior Analytics*, provided the only method for achieving "science." As he explains in his second chapter, the purpose of logic was to establish demonstrations.[79] The second principle is that Piccolomini's understanding of mathematics developed from Proclus, and Proclus's theory of mathematics essentially developed from Plato.[80] This second point of view is stated quite clearly at the beginning of the *Commentarium*, although the influence of it in his argument is not

[79] Piccolomini, *De commentarium*, p. LXXIIIIr.

[80] As will be demonstrated later, Piccolomini raised this aspect at the fundamental point in his argument in which he states the nature of mathematics.

made clear until later in the book. At the beginning of the work, after explaining that he no longer followed those from the medieval period, Piccolomini states,

> I chose to withhold [my opinion] until I learned that Proclus himself, feels exactly the same. With my soul filled with enormous joy, for having found such a rich witness, I have declared it loudly since then.[81]

In his presentation, Piccolomini indicates that the idea of the certitude of mathematics was a recent development of the medieval period that could not actually be found in other ancient authorities.[82] It is clear that involved in Piccolomini's criticism was the humanist criticism of the corrupted medieval understanding in contrast to a much purer interpretation provided by the ancient Greeks, especially Proclus.

Such a framework is certainly influenced by Piccolomini's active life as a Renaissance humanist. He was born in Sienna in 1508, and studied both mathematics and astronomy. He also wrote commentaries on many of Aristotle's texts, as well as commentaries in Latin and plays on various other subjects. He published the first vernacular logic textbook in Italian, and was an early advocate for the necessity of education for women.[83] Piccolomini was also active among the Italian academies, participating in two of them during his life. The first, the Accademia degli Intronati, was founded in 1525 in Sienna for the purpose of studying classical Greek and Roman texts, a devotion to which Piccolomini would remain for the entirety of his life.[84] In 1540 Piccolomini moved to Padua to study at the University of Padua, and while there

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[81] Piccolomini, *De commentarium*, p. LXXII: "eousque comprimendam duxi, donec Proclum ipsum, hoc idem sentire cognoscens, maxima animi laetitia affectus, testem tam locupletem nactus, id ipsum dehinc clara voce frequenter asserui."

[82] Among the Latin authorities he lists are Albertus, Thomas Aquinas, Marsilius, Egidius, Zimarra, Suessanus, and Acciaiolus.

[83] Daniele Cozzoli, "Alessandro Piccolomini and the Certitude of Mathematics," *History and Philosophy of Logic* 28, no. 2 (2007): p. 153.

[84] Ibid., p. 153

helped to start the Accademia degli Infiammati. The next year he was elected president of the Infiammati, and as president made it his goal to institute the study of classical texts at Padua.[85] In 1542 Piccolomini moved to Bologna in order to study with the natural philosopher Lodovico Boccadiferro. According to Daniele Cozzoli it is likely here that Piccolomini developed the idea that the methods of mathematics were inferior to those of physics. Additionally, since Boccadiferro was known as an Averroistic natural philosopher, it is likely also that Piccolomini's criticism of Averroes's understanding of mathematics developed while Piccolomini was in Padua. In 1544 Piccolomini returned to Sienna, teaching natural philosophy at a university there for a year. He then moved to Rome in 1546, where he remained until 1551. Throughout the 1560s and 1570s Piccolomini published often on a variety of topics. He died in Sienna in 1578.[86]

It was while in Rome, in 1547 that Alessandro Piccolomini published his *Commentarium de certitudine mathematicarum disciplinarum.* As mentioned, the framework within which Piccolomini organizes his argument is that of Aristotelian logic. The first three chapters are specifically devoted to establishing the value of logic itself as the superior form of knowledge. And the purpose of logic, as he explains, was to validate demonstrations. Such an understanding, Piccolomini points out, is based on the understanding of "all Greek interpreters."[87] In explaining the basic framework of logic Piccolomini presents three main types of demonstrations: the *quia, propter quid*,

[85] Ibid., p. 154.
[86] Ibid., p. 154.
[87] Piccolomini, *De commentarium*, p. LXXIIIIv

and the *potissima*.[88] In defining the relationship between the three types of demonstrations, Piccolomini lays out the basic framework that the *quia* gives the effect of a phenomenon and the *propter quid* the cause. According to Piccolomini, the *demonstratio potissima*, considered to be the highest form of certainty, was a type of demonstration which could give both the cause and the effect.[89]

However, it was Piccolomini's contention that mathematics could not fit the *demonstratio potissima*. Among the reasons he gave, the most extended treatment was with respect to the fact that mathematics could not treat any of the four types of Aristotelian causes. Mathematics did not fit the efficient cause because mathematics did not consider motion. Mathematics did not fit the final cause because a mathematical proposition was not clearly trying to prove a single syllogism and thus did not have a clear direction in which it was progressing.[90] Mathematics could not fit a material cause because it considered matter in the abstract.[91] The only one of the causes that Piccolomini gave any significant consideration was the formal cause, the one traditionally attributed to mathematics. However, as Piccolomini argues, not even this type of causation could fit. In order for mathematics to be a *demonstratio potissima*, the middle term of a syllogism had to be a definition of a subject or a definition of a property. As a result it was supposed to be unique to the particular subject. However, in a mathematical proof the same theorem may be proven in a variety of different ways,

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[88] Ibid., p. LXXXIIr. According to Piccolomini the first two develop from Aristotle's *Posterior Analytics*. The *potissima*, however, has its origin in the work of Averroes.

[89] Ibid., p. LXXXVr

[90] Ibid., p. CIIv-CIIIr

[91] Ibid., p. CIIIr-v

demonstrating that a particular proposition was not only connected to one particular subject in a causal way.[92]

To emphasize his point about the relationship between mathematics and Aristotle's causes, Piccolomini pointed toward Proclus's own description in his *Commentary on Euclid*, of Euclid's proposition I.32 mentioned above. Following Proclus's own characterization, he claimed that this example demonstrated the inability of a mathematical proof to be based on a causal demonstration. He then used this one proof to generalize about all Euclidean proofs, as well as those of Theodosius, Archimedes, or any other mathematics.[93] It becomes clear at this point that Piccolomini is not considering mathematics solely as it related to Euclid.

In formulating his argument, Piccolomini's point hinges on the way in which he frames the nature of mathematics. As Piccolomini argues, the most troubling aspect of connecting mathematics with the *demonstratio potissima* was the nature of mathematics itself, which prevented it from fitting the requirements of the middle term in a logical demonstration.[94] There are two predominant descriptions that Piccolomini gives to mathematics. One is "quantitates ipsae abstractae," the "the abstract quantity itself," and the other "quantum phantasiatum," the "imagined quantity."[95] Both terms are intended to emphasize the way in which the content of mathematics is separated from sensible matter. In explaining this point Piccolomini claims that he is drawing upon Proclus's *Commentary on the First Book of Euclid*, especially Book I. It is important to note that in the entirety of Piccolomini's argument, it is not until this pivotal chapter, chapter

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[92] Ibid., p. CV
[93] Piccolomini, *De commentarium*, p. CIIIv-CIIIIr
[94] Ibid., p. LXXXIIIv
[95] Ibid., p. LXXXXVIIr

eleven, that he invokes the authority of Proclus.[96] Thus, even though he had claimed Proclus's authority at the introduction of the book, the utility of Proclus for his argument does not occur until the middle of his book, when explaining the nature of mathematics.

What emerges from Piccolomini's characterization of Proclus, however, is the fact that he understood Proclus predominately as a follower of Plato. Piccolomini states,

> So Proclus derives from Plato the view that the mathematical things themselves, about which demonstrations are made, are sensible things neither altogether in a subject, nor entirely freed from it, but these mathematical figures are formed in the imagination, the occasion being afforded by quantities found in sensible matter. Moreover, the intellect derives those universal principles from these quantities that are in the imagination.[97]

Piccolomini is clear about how he understands mathematics. It is based on Proclus's own framework, which in its essence is something that had developed from Plato. What is interesting about Piccolomini making such a claim is that previously he had never connected Proclus and Plato together. While such a connection might have been something assumed during this time period, the emphasis, which he places upon it here, suggests that it is a connection that Piccolomini wants to be made clear. In his opinion, the nature of mathematics was Platonic.

In justifying the reason that a Euclidean Theorem could not fit the characteristics of the middle term Piccolomini clearly demonstrates his preference for the Aristotelian texts. Following his explanation of Euclid's proof I.32, he explains that all *demonstratio*

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[96] Aside from the introductory mention of Proclus stated at the beginning of this section, Proclus' authority is only mentioned a few times prior to this point, and they are all in lists with other agreed upon Greek authorities.

[97] Piccolomini, *De commentarium*, p. LXXXXVIIr-LXXXXVIIv: Concludit ergo Proclus ex Platone, quae res ipsae mathematicae, de quibus sunt demonstrationes, nec omnino in subiecto, sensibiles sunt, nec penitus ab ipso liberatae: sed in ipsa phantasia reperiuntur figurae illae mathematicae, habita tamen occasione a quantitatibus in materia sensibili repertis. Intellectus autem, ex iis, quae in phantasia sunt quantitatibus, rationes illas uniuersales colligit.

potissima have a middle term that is immediately connected to the effect. However, as he explains, mathematics could not achieve this. Organizing this argument into a syllogism, Piccolomini claims that the major premise, which stated that the middle term of a *demonstratio potissima* be the immediate cause of the effect, was established by Aristotle in the *Posterior Analytics*, and does not justify it beyond this.[98] The minor premise, that mathematics was an abstraction, could not fit this. It is noticeable throughout his explanations of the minor terms that he makes frequent recourse to explaining the nature of mathematics based on Proclus as well as Plato.[99] In short, Piccolomini's point is as follows. The Proclean mathematical system, largely based on that of Plato, could not fit the Aristotelian system of demonstration. Consequently, mathematics could not fit the Aristotelian system.

Nevertheless, even though Piccolomini did not believe that mathematics fit the form of an Aristotelian logical demonstration, this did not simultaneously imply that mathematics was of no benefit. Ironically enough, it was the abstractness of mathematics that contributed to its being the most easily understood subject. According to Piccolomini quantity is the most accessible accident of all matter, and the easiest to abstract from matter. As he explains, since the time of Aristotle and the Peripatetics, mathematics had been considered valuable because the clarity of its methods made it more accessible to students. Since mathematics treated quantity, the most sensible and clear quality, mathematics was the easiest perceived. Moreover, things are more easily understood in the abstract than according to their particular natures. Piccolomini is clear, though, that such a consideration is the result of the subject matter of mathematics

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[98] Ibid., p. CIIIIr
[99] Ibid., p. CIIIIr-CVr.

rather than the nature of mathematical demonstrations.[100] The main point for Piccolomini, however, is the level of knowledge that mathematics provided for an object. His conclusion was that it was the most basic type of knowledge. Thus, although Piccolomini had contended that mathematics was not a *demonstratio potissima*, he nevertheless believed that it still provided clear knowledge.

Historians have differed slightly as to the precise way in which to understand Piccolomini's text. According to Chikara Saski, Piccolomini is arguing that mathematical certainty is only applicable in a very few situations, whereas logic was applicable to a much wider array of knowledge, the most important of which was the *demonstratio potissima*.[101] Daniele Cozzoli explains that Piccolomini was trying to demonstrate the superiority of physics over mathematics within an Aristotelian system of demonstration.[102] Rivka Feldhay explains similarly, though she emphasizes more clearly that Piccolomini was trying to prevent mathematics from being understood as an autonomous science.[103] From this, it is clear that the historiography focuses on the way in which Piccolomini connected mathematics to logic. What it doesn't emphasize as clearly is the way in which Piccolomini worked to legitimize mathematics. Moreover, as indicated above, Piccolomini essentially bases his characterization of mathematics on that of Proclus, who he considers to be a follower of Plato. The picture that begins to emerge, then, is one in which the nature of a scientific demonstration develops from Aristotle and that of mathematics develops from Proclus and Plato. For Piccolomini this

[100] Ibid., p. CIX
[101] Saski, *Descartes Mathematical Thought*, p. 52.
[102] Cozzoli, "Alessandro Piccolomini and the Certitude of Mathematics," p. 168.
[103] Feldhay, "The use and abuse of mathematical entities," in *The Cambridge Companion to Galileo*, p. 84.

helps to explain the conflict that ensues between mathematics and natural philosophy. An Aristotelian classification system had a different framework for classification than that of a Proclean or Platonic.

It is clear from his *Commentarium* that Piccolomini was a committed Aristotelian. But, it is likely also the case that in the context of the development of sciences, he was interested in preserving the status of the philosopher, and consequently Aristotle himself. Daniele Cozzoli helpfully points toward some of Piccolomini's other works in order to elucidate Piccolomini's relationship between natural philosophy and logic. In 1551, Piccolomini published the first logical textbook in the Vernacular, *L'instrumento della filosofia*. In it Piccolomini expounded a framework for understanding the nature of logic and how it related to the practice of natural philosophy. He contended that, contrary to the ideal of the Aristotelian system, the physicist cannot penetrate into the essence of nature, but must draw the framework of nature from the way nature itself operates. Consequently, the natural philosopher does not proceed according to *demonstratio propter quid***,** although that is desirable, but according to *demonstratio quia*.[104] Yet, the reality, that *demonstratio propter quid* was rarely attainable, should not taint the striving for the goal. But equally true, it should be made clear that the theoretical clarity which mathematics appeared to provide should not be given a position of superiority over that of the physicist.

And it is likely Piccolomini's interest in distinguishing nature from theoretical, and natural from the unnatural, that motivated his *Commentarium*. What is often not recognized is that Piccolomini's *De certitudine* is attached to a Latin paraphrase of

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[104] Cozzoli, "Alessandro Piccolomini and the Certitude of Mathematics," p. 163-165.

Aristotle's *Mechanica*. According to Paul Rose and Stillman Drake, Aristotle's *Mechanica* was virtually unknown during the Medieval period, but experienced a significant revival during the sixteenth century by humanists who were interested in the precise recovery of Aristotle's corpus.[105] Moreover, as they contend, Piccolomini's paraphrase is best understood as falling into line with Piccolomini's humanist interest in recovering Aristotelian texts, similar to Piccolomini's commentaries on *Meteorology*, *Poetics*, and *Ethics.*[106] Moreover, in this commentary Piccolomini clearly indicates the way that mechanics was subalternate to geometry.[107] Consequently, his *De certitudine mathematicarum* is best understood as a further explication of the relationship between mathematics and mechanics. Taken together, the texts can be understood as a defense of Aristotle's natural philosophy.

In the Preface to both works, Piccolomini gives a traditional Aristotelian classification of the relationship between mechanics and mathematics, but states that mechanics is a mixed science that is subalternate to geometry. The purpose of geometry is to provide the mechanical arts with their causes and principles.[108] Moreover, it is likely that the reason Piccolomini wrote his *Commentarium* on mathematics was in order to preserve the distinctions that he was making between mechanics and natural philosophy. Whereas natural philosophy analyzes the causes of matter, mechanics treats the subject in a mathematical way.[109] His connection between mathematics and natural

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[105] Paul Rose and Stillman Drake, "The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture," *Studies in the Renaissance* 18 (1971): p. 67-69.

[106] Rose and Drake, "The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture," p. 82.

[107] Piccolomini, *De commentarium*, IIIr.

[108] Piccolomini, *In mechanicas*, p. IIIr.

[109] Ibid., p. 8v

philosophy may be most clearly seen when discussing motion, in Question 8 of his paraphrase of the *Mechanica*. Piccolomini states that natural bodies would not be able to touch the ground at only one point because their mass would not allow this. This is an obvious point for the natural philosopher, who considers objects as they naturally appear. However, the mathematician, in contrast, considers bodies apart from their mass, in the abstraction. For the mathematician it is not a problem for bodies to only touch the ground in one place. From this it is clear that, while mathematics is helpful for considering natural bodies, one has to consider sensory information beyond mere mathematical description.[110] Consequently, natural philosophy is superior. Taken together, then, it is clear that both the *Mechanica* and the *Commentarium* are intended to demonstrate the superiority of natural philosophy over that of mathematics.

Thus, in raising the question about the certitude of mathematics Piccolomini acts as a natural philosopher trying to retain the validity of his discipline.[111] In the way in which he framed the study of logic and mathematics, it becomes clear that mathematics would not fit. Convenient for his argument is the fact that mathematics was considered

[110] Ibid., 30v

[111] Such a consideration would fit this debate with the famous humanist Sperone Speroni, Piccolomini's presidential successor of the *Infiamatti*. After being elected, Speroni went about removing many of the traditional classical and philosophical elements of the society. For instance, he did away with the use of classical languages, and focused the educational instruction solely on the humanist disciplines. Speroni was focused on the practical disciplines, and consequently was not as interested in the speculative disciplines. According to Speroni, individuals should focus on the arts, literature, rhetoric, and poetry, and not the speculative disciplines of natural philosophy and metaphysics. Piccolomini, in contrast, was interested in preserving the validity of Aristotle's philosophy from the other disciplines. And, as his *Commentarium* indicates, the development of mathematics posed a challenge to it. See Heikki Mikkeli, "The Cultural Programmes of Alessandro Piccolomini and Sperone Speroni at the Paduan *Academia degli Infiammati* in the 1540s," in *Philosophy in the Sixteenth and Seventeenth Centuries*, p. 77-79.

in the abstract, a principle which Piccolomini argues is antithetical to the *demonstratio potissima*. However, in his analysis of Proclus, it becomes clear that Piccolomini has interpreted Proclus primarily as a follower of Plato, and consequently mathematics as essentially Platonic. This aspect becomes increasingly clear in the way in which Francesco Barozzi responds to Piccolomini.

FRANCESCO BAROZZI:
MATHEMATICS BETWEEN PLATO AND ARISTOTLE

Francesco Barozzi was Piccolomini's sharpest critic. In 1560 he published *Opusculum in quo una ratio, et duae quaestiones: altera de certitudine, et altera de medietate mathematicarum continentur*, which was intended as a direct counter to Piccolomini's framework. The same year that this work was published, he also published the first Latin translation of Proclus's *Commentary on the First Book of Euclid's Elements*. It is evident from both works that according to Barozzi a proper understanding of Proclus was necessary to help settle the issue about the certainty of mathematics.

Francesco Barozzi was a patrician, humanist, professor, and mathematician. He was born on the island of Crete in 1537 and was taught Latin and Greek at a young age and studied physics at the University of Padua. Not much else is know about his upbringing. In 1559 and 1560 he taught mathematics at the University of Padua, and then later helped to set up the Academia de' Vivi in Crete.[112] As Paul Rose notes, much of the remainder of his life was spent searching the libraries in and around Crete and acquiring Greek mathematical manuscripts.[113] In 1587, toward the end of his life, he was arrested and convicted by the Venetian Inquisition on charges of necromancy. Though it is not certain, it is suspected that this charge contributed to his inability to publish any other Greek works or Latin translations after this period. Nevertheless, in 1598, just a few years before his death in 1604, he wrote a letter to his nephew indicating his resilient belief in the power of education, and in particular the tremendous influence of the University of Padua.[114] It is evident, then, that Barozzi's involvement in the debate developed from his deep interests both in education as well as in the recovery, restoration, and dissemination of Greek texts.

When Barozzi published the *Opusculum* it was near the beginning of his professional career, having only previously published one book.[115] Barozzi's *Opusculum* is comprised of three sections. It begins with a transcript of his inaugural lecture at the University of Padua from 1559, in which he praises the divine guidance to his position as mathematics instructor, celebrates the dedicatee Daniel Barbero, a

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[112] Paul Rose, "A Venetian Patron and Mathematician of the 16th Century: Francesco Barozzi (1537-1604)," *Studi Veneziani* 1 (1977): p. 120-121. As Paul Rose notes, although it is documented that Barozzi was a teacher in 1559 and 1560 only, it is not certain whether or not he was paid for his work.

[113] In addition to Proclus, Barrozi recovered Greek manuscripts for Pappus, Hero of Byzantium, Archimedes, and he also wrote his own *Cosmographia* which he intended as a replacement of Sacrobosco's *Sphaera*. See Rose, "A Venetian Patron and Mathematician of the 16th Century," p. 119-178.

[114] Rose, "A Venetian Patron and Mathematician of the 16th Century," p. 119-122.

[115] His first work was a strictly philosophical work focused on the ordering of the sciences, *Oratio ad Philosophiam Virtutemque*, published in 1557. Rose, "A Venetian Patron and Mathematician of the 16th Century," p. 119-122.

wealthy Cardinal, and also celebrates the benefit of Proclus for the understanding of mathematics.[116] The two sections following, the *Quaestio de Certitude Mathematicarum* and the *Quaestio de Meditate Mathematicarum*, are specifically aimed toward the debate over the certitude of mathematics. Although Francesco Barozzi's *Opusculum* does not clearly mention Alessandro Piccolomini, historians are in agreement that Barozzi's text was intended to counter the claim of Piccolomini.[117] At the beginning he identifies that his reasons for considering mathematics to be in the first degree of certainty is in contrast to "clarissimus quidam" who historians have come to understand as Piccolomini.[118] After the publication of the work, it is known that Barbaro wrote a letter to Barozzi in which he expressed his hope that the *Opusculum* would demonstrate that Piccolomini's thesis had no historical foundation and was a mere modern novelty.[119]

But Barozzi's response to Piccolomini was not merely his *Opusculum*. A significant part was also his translation of Proclus's *Commentary on the First Book of Euclid*.[120] In the *Oratio*, Barozzi both praises Proclus as a mathematician and then explains that Barozzi had translated Proclus's work into Latin. As he explains, his

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[116] Barozzi, *Opusculum*, p. 3r-6r

[117] Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, p. 25-26; Feldhay, "The Use and Abuse of Mathematical Entities," in *The Cambridge Companion to Galileo*, p. 85-86; Sasaki, *Descartes's Mathematical Thought*, p. 54.

[118] Barozzi, *Opusculum*, 1560, p. 7r.

[119] Feldhay, "The use and abuse of mathematical entities: Galileo and the Jesuits revisited," in *The Cambridge Companion to Galileo*, p. 85.

[120] Barozzi, *Commentary on Euclid*. In the Preface to this translation Barozzi describes how he had discovered some Greek manuscripts of Proclus while on the island of Crete and that he had traveled about to restore these texts to a more authoritative complete text. The implication of this is that it was not based upon the 1533 Greek text of Simon Grynaeus, but instead was based on new textual evidence.

purpose in doing so was to make clear to all the benefit of Proclus's text. But, it wasn't simply that. Additionally, as he states, the benefit of Proclus's text is that it would enable one to better understand both Plato and Aristotle's texts.[121] Thus, according to Barrozi a response to Piccolomini, involved an understanding of Proclus as well as the relationship between Aristotle and Plato on the status of mathematics.

It is the two other sections, though, that specifically respond to Piccolomini's text. At the beginning of the second section, the "Quaestio de certitudine mathematicarum," Barozzi identifies what he considers to be a contentious passage from someone who recently had questioned the certainty of mathematics in a text published alongside Aristotle's *Mechanica.*[122] The first half of this work is solely a detailed summary of the reasons Piccolomini had given as to why mathematics did not fit a *demonstratio potissima.*[123] Based upon the accuracy with which Barozzi responds to Piccolomini it is evident that he was very familiar with the entirety of Piccolomini's text.

What is interesting in this second section is the way in which Barozzi frames his response to Piccolomini. Rather than specifically address the nature of the *demonstratio potissima*, Barozzi instead focuses on clarifying the nature of mathematics itself. According to Barozzi mathematical objects themselves are abstractions, which makes them the most comprehensible and therefore the most easily discernible.[124] For Barozzi, the consequence of mathematics being so easily discernible is that a mathematical proof

[121] Barozzi, *Opusculum*, p. 4v-5r
[122] Ibid., p. 7r.
[123] Ibid., p. 7v-10v.
[124] Ibid., p. 11r

could then be considered a *demonstratio potissima*.[125] Thus, it is noticeable that, in contrast to Piccolomini who had focused on the inability of mathematics to achieve a *demonstratio potissima*, Barozzi instead focused on the clarity of the methods of mathematics. Consequently, it increasingly becomes evident that Barozzi and Piccolomini had two different understandings of the nature of mathematics.

Such a difference may be seen in the way in which they differed as to the proper understanding of Proclus. This can be noticed in the way in which Barozzi addresses Euclid's proof I.32, which Piccolomini had used to demonstrate the inability of Euclid's theorems to fit one of Aristotle's causes. While Barozzi conceded the fact, as Proclus had done, that mathematics could not treat causes in all situations, he also pointed out, as Proclus had done, that it could in some situations. The example he used was the one mentioned above in which equilateral triangles may be causally constructed from equal circles.[126] Thus, an important component to Barozzi's response to Piccolomini was a correction to Piccolomini's reading of Proclus.

In addition to this Barozzi also outlines the way in which his opinion that the certainty of mathematics implies its position as a *potissima*, fits with that of Plato, Aristotle, Proclus, Simplicius, Themistius, Eustrates, and Averroes. The most prominent of these were Plato, Aristotle, and Proclus. Beginning with Plato, Barozzi cites Plato's *Republic* VII as well as the *Phaedrus.* Of these two, Plato's *Republic* VII was particularly important for the doctrine of mathematics as it explained that mathematics was the discipline that would help one to progress to the higher realms of dialectics. Although he doesn't give a detailed explanation, from these texts Barozzi explains that

[125] Ibid., p. 12v.
[126] Ibid., p. 24v.

it can be deduced that Plato believed that mathematics could “properly be called a demonstration” and as a result was certain.[127] Throughout all of Barozzi’s text, it is clear that Plato’s foundation of mathematics, although different than Aristotle's, did not preclude the usefulness and applicability of mathematics.

Following this Barozzi transitioned to an analysis of Aristotle. His main purpose in this section is to demonstrate that Aristotle indicates in the *Posterior Analytics* the fact that mathematics is able to identify the causes, and consequently able to be classified as a *propter quid*. In making his explanation Barozzi specifically mentions examples from the mixed sciences, such as the relationship between the physician and geometer. The physician has the knowledge of why a wound heals slowly, but the geometer can explain this *propter quid*.[128] When Barozzi explains Proclus’s opinion he deviates from the way Piccolomini had referred to Proclus. Whereas Piccolomini had referred to Proclus in order to legitimate the fact that mathematics was an abstraction, Barozzi refers to Proclus in order to demonstrate the relationship between the speculative sciences. Barozzi, following Proclus’s prologue, argues that mathematics is more certain that natural philosophy, but less certain than divine science.[129] From this it becomes apparent that a significant part of what Barozzi was trying to accomplish was the reconciliation of Aristotle's classification of the sciences with that of Plato.

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[127] Ibid., p. 13r-13v.

[128] Ibid., p. 14r. Cf. *Posterior Analytics* I.13.

[129] Barozzi, *Opusculum*, p. 14v-15r. Such a comparison of the disciplines is important for his appeal to the certainty of mathematics, especially because of the way he compares them at then end of this Quaestio. According to Barozzi divine science is considered to be certain not because of its demonstrations or material nature, but because of the nature of its subject. Similarly, mathematics should not be considered uncertain due to its immateriality or its inability to be shown through demonstration. See Barozzi, *Opusculum*, p. 32r-32v.

When he finally concludes that mathematics could be used to demonstrate the cause, and consequently *propter quid*, he suggests that such an opinion is found in Aristotle, Aristotle's commentators, Proclus, and also Plato, once again noting *Republic* VII.[130] The effect of this was that Barozzi was indicating that Aristotle was essentially in agreement with the prevailing doctrine of mathematics, found in Plato. What begins to emerge from Barozzi's treatment of these three authorities is the fact that Proclus is able to help reconcile the differences between Plato and Aristotle. In the first section of the *Opusculum*, the transaction of his speech upon becoming mathematics instructor at Padua, he alludes to this fact. He states that it is his hope that his translation of Proclus would make the "opinions" of others, especially Plato and Aristotle, clearer to future generations.[131]

Such reconciliation becomes clearer in the final treatise of his *Opusculum*, the "Quaestio de medietate mathematicarum." In it he is trying to determine the differences in perspective among various individuals, such as Plato, Aristotle, and various Peripatetics. He begins it by detailing a brief history of the classification of the sciences, particularly noting the role of Boethius, Simplicius, and Ammonius within it.[132] However, it is clear from the text that his biggest interest is in the reconciliation of Plato and Aristotle together. He writes,

> Plato and his school prefer mathematics over natural science. On the other hand, Aristotle and all Peripatetics have said that natural science is prior to

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[130] Barozzi, *Opusculum*, p. 31r.

[131] Barozzi, *Opusculum*, p. 5v: "totum vt id Procli volumen interpretari possim, quamplurimas Platonis, & Aristotelis, aliorumque autorum sententias, quae ad mathematicam spectant disciplinam, multis familiarissimas in posterum futuras polliceor."

[132] Barozzi, *Opusculum*, p. 35r-36v

> mathematics, and [even] they have granted it an intermediate position between natural and divine science.[133]

Although he admits that these opinions seem contradictory, in reality they are the same. The difference, as he indicates, is one of perspective. Whereas Plato had organized the disciplines in accordance with nature, thereby presenting divine science as first, Aristotle had organized the disciplines in accordance with human perspective. The consequence of this is that Aristotle had placed natural science first, since it was the science that was most easily comprehended by the human mind.[134] Moreover, according to Barozzi the difference between Plato and Aristotle was one of perspective, as each sought to explain the significance of the sciences based upon their own framework for the classification of the sciences.

Barozzi is clear that a response to Piccolomini would involve an understanding of the nature of mathematics as a middle discipline. In so doing he is invoking a much larger conversation on the classification of the sciences. In addition, though, Barozzi also makes clear that a significant component to this debate was a proper realization of the differences between the way Plato understood mathematics and Aristotle did. Such a difference would continue to be reiterated as the debate continued in the Jesuit Order between Christoph Clavius and Benito Pereira.

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[133] Barozzi, *Opusculum*, p. 37r: "Plato .n. eiusqúe sectatores mathematicam naturali praeposuere. Arist. autem, omnesqúe peripatetici naturalem mathematica priorem esse dixerunt, ipsiqúe medium inter naturalem, & diuinam locum tribuerunt."

[134] Ibid., p. 40r.

MATHEMATICS IN THE JESUIT ORDER: CLAVIUS AND PEREIRA

One of the starkest ironies of the Jesuit Order is that they developed one of the strongest, if not the strongest, mathematics program in all of Europe during a time period in which mathematics was not given much recognition.[135] Moreover, as mentioned in the Introduction, because of this it becomes all the more interesting to consider the way in which the sixteenth-century Jesuits appropriated the debate over the certainty of mathematics. The Spanish natural philosopher, Benito Pereira, and the German astronomer, Christoph Clavius, were both teachers at the Collegio Romano in the time period in which the certainty of mathematics was being debated. It is clear from both of their works that they were deeply engaged in the sixteenth-century Italian discourse over mathematics. More specifically, the picture that begins to emerge is one in which Pereira is slightly modifying Piccolomini and Clavius is slightly modifying Barozzi, each to fit their particular situations.

What becomes different about the context of Pereira and Clavius, as compared to Piccolomini and Barozzi, however, is the fact that their controversy more clearly involved institutional dynamics. The Collegio Romano had just formed, and the task that was placed before them was the establishment of an educational curriculum. In

[135] See Peter Dear, "Jesuit Mathematical Science and the Reconstitution of Experience in the Early Seventeenth Century," *Studies in History and Philosophy of Science Part A* 18, no. 2 (1987): p. 133-175; John O'Malley, "The Historiography of the Society of Jesus," in *The Jesuits: Cultures, Sciences, and the Arts 1540-1773*, edited by John O'Malley, Gauvin Bailey, Steven Harris, and T. Frank Kennedy (Toronto: University of Toronto Press, 1999), p. 32.

1560, twenty years after the Jesuits had been established as an official order, Ignatius's secretary, Polanco wrote a letter to all the superiors of the Order. In it he stated,

> Generally speaking, there are [in the Society] two ways of helping our neighbors: one in the colleges through the education of youth in letters, learning, and Christian life, and the second in every place to help every kind of person through sermons, confessions, and the other means that accord with our customary way of proceeding.[136]

Polanco's speech here indicates the way in which the Jesuits were institutionalizing education, and placing it in a position of significance within the Society.

MATHEMATICS IN THE COLLEGIO ROMANO

Initially Ignatius, and the other founders of the Jesuit Order, had defined their purpose in their charter, *Formula of the Institute*, as "the propagation of the faith and the progress of souls in Christian life and doctrine."[137] They perceived of their work as that of missionaries, following in the footsteps of the Christian Apostle, Saint Paul. They were not to earn an income, but were to live off of charity. Yet, by 1550 Ignatius and the other Jesuit leaders had decided that one of the ways in which they could advance their ministry was through establishing schools. Thus, after a decade of existence they had substituted the ideals of itinerant preaching with the permanence of schools. By 1580 the Jesuits had 144 schools, around 450 in 1630, and 850 by the

[136] Quoted in John O'Malley, *The First Jesuits* (Cambridge, MA: Harvard University Press, 1993), p. 200.

[137] Quoted in John O'Malley, *The First Jesuits*, p. 5. As O'Malley points out, what is interesting is that when the Jesuits were officially recognized by the Catholic Church in 1550 this was changed to "the defense and propagation of the faith," likely reflecting the doctrinal insistence of the Counter Reformation.

middle of the eighteenth century.[138] By the seventeenth century in most of these schools the instruction of mathematics had become of central importance.[139]

The earliest proponent of mathematics within the order was Hyeronimo Nadal, the first mathematics instructor at Messina, the first Jesuit school. Nadal had received his mathematics training at the Sorbonne in Paris and was the first to organize a mathematics curriculum, doing so in 1548 for the purposes of teaching at Messina. In it he included a few fundamental points that were essential for further developing the mathematical aspects of astronomy, natural philosophy, and metaphysics.[140] Impressed with the success of his early mathematics curriculum, Nadal composed a more formal mathematics curriculum to be inserted into the *Constitutions*. Much more focused than the first curriculum, this one was focused on integrating the basic framework of mathematics necessary for understanding Aristotelian themes over the course of three years. However, this course was not adopted within the Society. Instead what was adopted was the mathematics program from the Collegio, in which mathematics courses were supplementary to physics courses.[141] The effect of this is that rather than establishing mathematics as a fundamental subject, as Nadal had desired, mathematics took on a secondary role. At the same time, the preference given to the Collegio

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[138] Mordechai Feingold, "Preface," in *Jesuit Science and the Republic of Letters*, edited by Mordechai Feingold (London: MIT Press, 2003), p. vii.

[139] The best description of this is Antonella Romano, *La Contre-Reforme mathematique: Constitution et diffusion d'une culture mathematique jesuits a la Renaissance (1540-1640)* (Rome: Ecole Francaise, 1999). But also see Chikara Sasaki, *Descartes's Mathematical Though*, p. 30-44.

[140] Romano Gatto, "Christoph Clavius' 'Ordo Servandus in Addiscendis Disciplinis Mathematics' and the Teaching of Mathematics in Jesuit Colleges at the Beginning of the Modern Era," *Science & Education* 15 (2006): p. 236.

[141] Ibid., p. 237.

Romano was a good indication of the importance placed upon the Collegio Romano in establishing the educational pattern for the wider Jesuit Order.

The first mathematician at the Collegio Romano was Baldassar Torres, from 1553-1561, who was a physician that intermittently taught mathematics. His most concerted efforts to institute a mathematics course was from 1557-1560, in which he organized two successive years of mathematics courses which would be taught alongside logic and physics courses. The first year was devoted to practical mathematics and arithmetic, and the second year to planetary theory, the use of the astrolabe, the theory of optics, and other geometrical concepts. Although Clavius was not at the Collegio Romano during the time period in which these two courses were taught, Romano Gatto has found correspondence between Clavius and Torres, indicating that Clavius took an active role in the organization of the mathematics curriculum at the Collegio Romano from the beginning.[142]

At the same time period in which Torres, with the help of Clavius, was developing the mathematics courses at the Collegio Romano, Benito Pereira was organizing the Logic, Metaphysics, and Physics courses. Based upon his archival research of Pereira's lecture notes, however, Richard Blum has indicated that Pereira's work which combats the status of mathematics, the *De Communibus omnium rerum naturalium principiis et affectionibus*, published in 1576, was largely comprised of his lecture notes from these classes in the 1560's.[143] Thus, the context of the debate between Pereira and Clavius was more than strictly curricular or disciplinary. Due to their

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[142] Ibid., p. 237-239.

[143] Richard Blum, "Benedict Pererius: Renaissance Culture at the Origins of Jesuit Science," *Science & Education* 15 (2006): p. 280.

positions in the Collegio Romano, and the early importance of the Collegio Romano in the development of the Jesuits, its importance was connected to the larger development of the Collegio Romano which had implications on the wider development of the Jesuits.

BENITO PEREIRA:
MATHEMATICS ARE ESSENTIALLY PLATONIC

Benito Pereira was a Spanish Jesuit who lived from 1535-1610, and who taught at the Collegio Romano from 1558-1610. At the beginning of his career he taught Physics, Metaphysics, and Logic.[144] In 1576 Pereria became a lecturer in theology, and would remain in that position until his death, writing many commentaries on the Bible in the process.[145] The *De principiis*, published in 1576, went through at least fourteen different printings throughout the latter half of the sixteenth century and the first two decades of the seventeenth century.[146] The influence of this work was extensive. It was

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[144] A. C. Crombie, "Mathematics and Platonism in the Sixteenth-Century Italian Universities and in Jesuit Educational Policy," in *Science, Art and Nature in Medieval and Modern Thought* (London: The Hambledon Press, 1996), p. 133.

[145] Crombie, "Mathematics and Platonism in the Sixteenth-Century Italian Universities and in Jesuit Educational Policy," p. 133. Of his commentaries, the most well-known commentary was the one on *Genesis*, which developed a hermeneutical approach that was influential for Galileo in his *Letter to the Grand Duchess Christina*. See, Richard J. Blackwell, *Galileo, Bellarmine, and the Bible* (Notre Dame: University of Notre Dame Press, 1991), p. 20-22.

[146] Charles Lohr, *Latin Aristotle Commentaries: Book II: Renaissance Authors* (Firenze: Leo S. Olschki Editore, 1988), p. 318. It should be noted that within the historiography there is a 1562 edition that is frequently listed. The origination of this dates back to early work on the Jesuits by Sommervogel. However, no modern scholars have been able to find it. See Richard Blum, *Studies on Early Modern Aristotelianism*, p. 140, n. 3.

formative in the development of Jesuit education, particularly the school at Coimbra, and was known as a trustworthy textbook for the history of philosophy.[147]

From the beginning of his time in the Jesuit Order Pereira was interested in education. One of the earliest formulations of Jesuit educational philosophy was actually by Benito Pereira. In 1564, Pereira wrote a treatise, the *Ratio studendi*, to his fellow professors at the Collegio Romano. John O'Malley has characterized this document as a synthesis of classical, medieval, and Renaissance ideas about education, and one that shows particular preference toward the authority of Aristotle. In this document Pereira also very clearly indicates that the purpose of education is the formation of a person's opinions.[148] As a result of such a desire to shape individuals' intellectual framework, Pereira strongly emphasized the need to consider the opinions of others, particularly in situations that did not clearly pertain to matters of the faith. It was only through such a process that one would be able to discover truth. Moreover, as Feldhay contends, Pereira's educational philosophy was one that encouraged the challenging of ancient opinion to contribute to the formation of proper thinking.[149]

This certainly fits with the way Pereira begins the *De principiis*. He writes,

> I give much credit to Plato, even more to Aristotle, but most to reason…Whatever I see in Aristotle as convenient and consistent, I take as probable. But what appears as coherent with reason I judge as true and certain.[150]

[147] Feldhay, "The use and abuse of mathematical entities," in *The Cambridge Companion to Galileo*, p. 92.
[148] O'Malley, *The First Jesuits*, p. 214.
[149] Feldhay, *Galileo and the Church: Political Inquisition or Critical Dialogue* (Cambridge: Cambridge University Press, 1995), p. 138-139.
[150] Pereira, "Praefatio," in *De principiis*, p. 4: "Ego multum Platoni tribuo, plus Aristoteli, sed rationi, plurimum."

In his study of early modern Aristotelian commentaries, Richard Blum comments that this aspect of Pereira, his interest in developing a reasoned approach to philosophy, is what helps situate him as a Renaissance intellectual rather than a medieval one. As he points out, what is particularly interesting about Pereira's argument is that, although he provides certain authorities which ought to be followed, most notably Albertus and Thomas Aquinas, Pereira also upholds the necessity of proper philosophical thinking.[151] Consequently, for Pereira, there is a proper way in which to approach subjects. This will become apparent when considering the way he reasoned his way through mathematics.

Before addressing Pereira's understanding of mathematics it is necessary to mention one historiographical issue. In some of the historiography, Pereira is listed as an Averroist. The reason for this is that in 1567 the German Jesuit Petrus Canisius wrote a letter to the General of the Jesuits, Francesco Borgia, stating that students were leaving Rome as followers of Averroes. Since Pereira was the prominent natural philosopher in Rome during this time period, historians have come to understand him as a likely source of such instruction.[152] Based upon his study of Pereira's lecture notes as well as some private correspondence, Richard Blum notes that it is evident that Pereira did in fact believe that one ought to teach Averroes. In the early 1560s Pereira composed a document listing the authorities that should be taught. While Aristotle, Plato, and many of the Peripatetics were listed in this document, it is also evident that

[151] Blum, *Studies on Early Modern Aristotelianism*, p. 149-150. In Blum's opinion Pereira is operating from a similar framework as someone like Pomponazzi.

[152] For an example of this historiographical opinion, see Craig Martin, *Subverting Aristotle* (Baltimore: Johns Hopkins University Press, 2014), p. 89-90.

Averroes was a prominent authority as well.[153] However, as Blum also indicates, Pereira specifically taught against the idea of the intellective soul, the specific aspect of Averroes that the Jesuits found troubling.[154] Blum also provides a compelling case that the accusations of Pereira as an Averroist could have developed from a dispute between Pereira and another Jesuit.[155] Nevertheless, fitting with his insistence on reasoning through various authorities, it is clear that Pereira did not shy away from engaging the texts of Averroes.

The text of *De principiis* is best understood as a philosophy textbook for the time period. Richard Blum notes that the title of Pereira's work suggests that it ought to be understood as similar to other Renaissance philosophy texts, such as Telesio, Pompanazzi, or Portius, all of whom sought to do the same thing that Pereira did, provide universal principles from which everything could be explained.[156] Moreover, Pereira is clear from the beginning of his work that he intends to frame the text in the context of the classification of the sciences. At the beginning of Book V he explains the traditional tripartite division of the speculative sciences, and then works through a variety of the scholastic interpretations of them. From this it is evident that Pereira is most interested in outlining the organization of metaphysics, which he notes is also

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[153] Blum, *Studies on Early Modern Aristotelianism*, p. 142-45. Blum also notices in some of Pereira's lecture notes an interest in presenting Averroes as a text to be taught.

[154] Ibid., p. 146.

[155] Ibid., p. 147. One of the best pieces of evidence that Blum provides is the fact that one of the students who Canisius had listed as an Averroist is not listed as having studied with Pereira while at the Collegio Romano.

[156] Richard Blum, "Benedictus Pererius: Renaissance Culture at the Origins of Jesuit Science," p. 279.

referred to as the divine science as well as the universal science.[157] As Richard Blum notes, part of what Pereira is trying to do in this section is avoid what he perceives as the theological issues with the way metaphysics had traditionally been understood.[158] Once he establishes metaphysics as the universal science, then for Pereira, it naturally follows that mathematics and physics are both subalternate to metaphysics.[159] Mathematics specifically treats of quantity, which he explains is an "accidental" feature, and physics treats of bodies in motion, which he classifies as addressing the "substance." The reason mathematics is placed in the middle of the speculative sciences is that its characterization of abstractness enables one to understand the divine. As he points out, this point can be demonstrated based on Alexander, Averroes, and Proclus.[160]

Following this, Pereira remarks on the nature of mathematics. Such was the context in which Pereira framed the significance of mathematics. It was a framework developed out of a broader context on the classification of the speculative sciences, and one in which he had already described mathematics as pertaining both to abstraction and accidental features. In this section his main point is that mathematics should not properly be considered a science.[161] According to Pereira "scire" is the knowledge of a thing according to its cause, and "scientia" is a demonstration of effect. Moreover, for a demonstration to be the most perfect type, which Pereira here likely means a *demonstratio potissima*, the demonstration had to be "per se," or "according to the

[157] Pereira, *De principiis*, p. 15.
[158] Blum, *Studies on Early Modern Aristotelianism*, p. 151-154.
[159] Pereira, *De principiis*, p. 23.
[160] Ibid., p. 23-24.
[161] Ibid., p. 24: "Mea opinio est, Mathematicas disciplinas non esse proprié scientias."

thing," rather than "per accidentia," or "according to accident." However, according to Pereira, mathematics only considers "quantity" and consequently does not consider "per se" but "per accidentia."[162]

What is most interesting about Pereira's analysis is the way in which he appeals to textual authorities to prove his point. Pereira organizes his argument into a syllogism, in which each part has its own textual legitimization. According to Pereira, the major premise, that knowledge develops from an explanation of causes, was based upon Aristotle's *Posterior Analytics*.[163] However, aside from identifying that it was based on Aristotle's *Posterior Analytics*, Pereira does not provide any further legitimation. In contrast, the minor premise, which states that mathematics does not treat quantity nor operate according to causation, has a much more extended explanation. Pereira claims that this premise is justified based upon Book VII of Plato's *Republic*. Pereira focuses on the fact that mathematicians "dream" about quantity, suggesting that mathematics was a non-material framework. It proceeded not from knowledge of the sensible world, but from certain suppositions. And, it was because of this that mathematics could not be considered a proper science.[164]

Furthermore, according to Pereira, Proclus in his *Commentary on the First Book of Euclid* had explained such an opinion.[165] After this Pereira included the same criticism that Piccolomini had presented, Euclid's proof I.32, in which a triangle with an extended side is considered. In his explanation, as Piccolomini had done, he points toward Proclus's explanation that this proof demonstrates the fact that mathematics

[162] Ibid., p. 24.
[163] Ibid., p. 24.
[164] Ibid., p. 24.
[165] Ibid., p. 24.

cannot be considered causal. Moreover, also similar to Piccolomini, Pereira considers this proof to be representative of all geometrical proofs. Pereira's conclusion, though, is much more adamant than that of Piccolomini's. Pereira states that the reason the external angle could not causally be related to the internal angles is because the external angle is an "accident" to the internal angles.[166] Very similar to Piccolomini's argument, what emerges from Pereira's reasoning through the relationship of mathematics with Aristotelian demonstration is a divide between an Aristotelian understanding of demonstration and a Platonic framework for understanding mathematics.

Later on in his *De principiis*, Pereira specifically addressed the relationship between mathematics and a *demonstratio potissima*. In the section he repeats many of the same criticisms raised by Piccolomini as to why mathematics should not be considered a *demonstratio potissima*. His main point was that mathematics could not treat the four Aristotelian causes.[167] In addition, similar to Piccolomini, Pereira states the benefit of mathematics is the ease with which it can be understood. He states that it is the "most certain, most evident, and easiest," because its subject matter, "quantity" is easily perceived by all the senses. Pereira suggests that one be taught the subjects of geometry and arithmetic so that one could best approach the mixed sciences.[168] Though it should not be considered a science, it was nevertheless valuable for developing knowledge of the other sciences.

[166] Ibid., p. 24: "anguli externi esse impossibilem, nihilominus tamen illa passio inesset triangul; at, quid aliud definitur esse accidens quám quod potest adesse & adesse rei praeter eius corruptionem?"
[167] Pereira, *De principiis*, p. 72-73.
[168] Ibid., p. 72-73.

Rivka Feldhay has pointed out a fundamental difference between Pereira and Piccolomini that helps one understand the way in which Pereira understood mathematics. As Feldhay notes, Pereira rejects Piccolomini's idea of mathematics as a "medietas." When Piccolomini had borrowed this concept from Proclus, he intended to use it to justify the abstracted nature of mathematics from sensible matter. However, for Pereira, the "medietas" of mathematics evoked the Platonic epistemology of the sciences. It was connected to Plato's doctrine of reminiscence, the idea that dialectical reasoning enables one to arrive at the understanding of the ideal forms. For Pereira, who clearly desired to uphold Aristotelian ideology and as well as Catholic doctrine, this was problematic. The way in which Pereira is able to reject this doctrine is to question the necessity of sense experiences. If reminiscence was true, as Pereira wonders, why would individuals have actual sense experiences? However, because sense experiences are true, then, the idea of reminiscence must be rejected.[169]

Recently, Richard Blum has compellingly argued that not only was Pereira's *De principiis* intending to present a framework for education, but that it was also intended to engage the variety of opinions that existed in Renaissance Italy at the time period, of which Platonism was a particularly prominent group.[170] A particularly compelling observation that Blum makes toward this point is in the section in which Pereira denies that mathematics is a science. At the end of this section Pereira includes an extended

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[169] Feldhay, "The Use and Abuse of Mathematical Entities," in *The Cambridge Companion to Galileo*, p. 92-93; cf. Pereira, *De principiis*, p. 80.

[170] Blum, *Studies on Early Modern Aristotelianism*, p. 159.

quotation from Plato's *Republic*, Book VII.[171] The context of the quotation is about the Platonic process of dialectic, and explains that dialectic was the only process which could actually arrive at first principals. After providing the lengthy quotation, Pereira parenthetically adds "intelligit autem primam Philosophiam." This indicates that in Pereira's mind, his framework of metaphysics, which he also refers to as first philosophy, encompasses that of dialectics within Platonic philosophy. As Blum contends, what Pereira likely means by this is that that Platonic dialectics, which was beginning to flourish in Renaissance Italy at the time in which Pereira taught and wrote this work, was subsumed within Pereira's concept of metaphysics.[172]

Such a concern for Platonic ideology indicates that, for Pereira, the framework of mathematics was closely connected with platonic ideology. Similar to Piccolomini, Pereira had fundamentally considered the nature of mathematics, that it treated quantity in the abstract, antithetical to the nature of demonstrative causation. However, more so than Piccolomini, Pereira's rejection of mathematics appears to be also connected to his fear of platonic ideology. Such a difference of emphases would make sense. Piccolomini, a devoted classicist and logician, was most interested in preventing mathematics from being considered useful to logic. Whereas, Pereira, a member of a religious order, and thus with presumed theological commitments, approached the subject differently.

At the same time period in which Pereira was formulating his philosophical framework, and advocating that mathematics not be considered a science, Christoph

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[171] Based on an analysis of this quotation of Plato, Blum argues that this is likely quoted from Ficino's translation, Blum, *Studies on Early Modern Aristotelianism*, p. 158.

[172] Blum, *Studies on Early Modern Aristotelianism*, p. 159.

Clavius was advocating for the usefulness of mathematics. Moreover, if there were any theological concerns about the association of mathematics with platonic ideology, Clavius would dispel them all with the way in which he advocated for their use.

CHRISTOPH CLAVIUS:
THE WIDESPREAD UTILITY OF MATHEMATICS

Among historians it is common to emphasize the role of Christoph Clavius in solidifying the role of mathematics among the Jesuits. James Lattis has made this point most comprehensively in *Between Copernicus and Galileo*, by arguing that a significant amount of Clavius's success was due to his *Commentary on Sacrobosco*.[173] Published in several editions from 1570 to 1611, according to Lattis this text became Clavius's principle tool for teaching mathematics. Moreover, it is clear in Lattis's work that he is most interested in understanding how a committed Ptolemaic astronomer tried to respond to such criticisms as the nova of 1572, the comet of 1577, the novas of 1600 and 1604, and Galileo's telescopic discoveries.[174] But, it was also during this time that Clavius outlined his mathematics curriculum for the Jesuits and published his commentary on Euclid's *Elements*. It is likely that a significant goal of Clavius was the practical aspect of aggrandizing the status of the mathematicians within the education

[173] James M. Lattis, *Between Copernicus and Galileo: Christoph Clavius and the Collapse of Ptolemaic Cosmology* (Chicago: University of Chicago Press, 1994).

[174] Ibid., p. 145-179 for a helpful synthesis of these strains on Ptolemaic cosmology and Clavius's activity.

structure, especially since it is clear that during the Renaissance in Italy, within the university system, mathematicians were of low social status.[175]

Christoph Clavius was born in 1538 in Bamberg, Germany.[176] Until the time he officially entered the order very little is specifically known about the influences on his life. It is likely that he became aware of and joined the Jesuit Order as a result of the work of one of the Jesuit missionaries in Germany. He was received into the Order in 1555, after which Clavius was sent to the University of Coimbra, in Portugal, where he enrolled as a student. It is documented that Clavius studied both the humanities and philosophy while at Coimbra, though it is not documented that he specifically studied mathematics. He remained in Coimbra until 1560, at which time he returned to study at the Collegio Romano. In 1561 he is listed as studying physics and metaphysics at the Collegio Romano, and by 1562 he had progressed to the study of theology. He was ordained in 1564, and professed his vows in 1575, becoming a full Jesuit member at that time.[177]

Clavius is first listed as a mathematics instructor at the Collegio Romano in 1567, though it is likely that he had begun teaching there as early as 1563.[178] One of the most significant questions in understanding Clavius's development is determining how Clavius became involved in mathematics. Since Clavius received his education in a time period in which mathematics was developing, historians have tried to determine

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[175] Ibid., p. 32. On the status of mathematicians during this time period, see Mario Biagioli, "The Social Status of Italian Mathematicians, 1450-1600," *History of Science* 27 (1989): p. 41-95.

[176] Ibid., p. 12. Though as Lattis points out, there is some discrepancy as to his precise birth, with some records indicating that he was born in 1537.

[177] Ibid., p. 13-14.

[178] Ibid., p. 20. The date of 1563 is based on Lattis's observation that in 1584 Clavius had commented that he had been teaching mathematics for twenty years.

the way in which Clavius's mathematical abilities developed. It is not likely that the region where he grew up in Germany had an active mathematics program.[179] Perhaps a better suggestion is the influence of Pedro Nuñez, a famous mathematician and cartographer who occasionally taught at Coimbra. Pedero Nuñez wrote about many important contemporary astronomical works, such as Sacrobosco's *Sphaera* and Peurbach's *Theoricae novae planetarium*. However, according to the academic records from Coimbra, Nuñez is not actually listed as one of Clavius's instructors.[180] Although later in his life Clavius would mention that Nunez was one of the greatest mathematicians he had ever met, it is not clear the extent to which he had actually interacted with Nuñez.[181]

While Clavius does record an eclipse in his *Sphaera* that he could only have observed while at Coimbra, thereby demonstrating his activity in astronomy, it is not clear the extent to which such interests developed from school or from personal pursuits.[182] Consequently it is probable that a significant amount of Clavius's development in mathematics was on account of personal interest. James Lattis notes that Baldi, Clavius's biographer, cast Clavius as a humanist being educated in the style of the ancient Greeks. Baldi wrote, "Desiring then to understand [mathematics] well, he set himself to exhaust the subject on his own without any teacher's help, so that in this discipline he affirms himself to be, as the Greeks say, an autodidact."[183] And, as Baldi

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[179] Although as Lattis points out, one of the first Astrolabe manuals comes from Clavius's region in Bramberg. See Lattis, *Between Copernicus and Galileo*, p. 15-16.
[180] Ibid., p. 15.
[181] Ibid., p. 14
[182] Ibid., p. 15.
[183] Quoted in Lattis, *Between Copernicus and Galileo*, p. 17.

indicates, it was Clavius's study of the *Posterior Analytics* that contributed most significantly to his interest in mathematics.[184]

Clavius's Prolegomena to his commentary on Euclid contains his addition to the debate over the certainty of mathematics. Barozzi's deep influence on Clavius is evident from the Prolegomena. In it Clavius credits Barozzi as being a significant influence on his development, particularly due to Barozzi's excellent translation of Proclus.[185] His engagement with the debate occurs in the section "The Nobility and Preeminence of the Mathematical Sciences." He first begins by explaining the position of mathematics within the classification of the sciences, relative to both metaphysics and physics. As he explains, the nature of mathematics itself warrants its position between the two disciplines. In his opinion, since metaphysics was separated from matter in both reality and theory and physics was connected to matter in both reality and theory, then the middle position of mathematics made sense because it "treats things which are considered separate from any matter."[186] Such an explanation closely mirrors the approach that Barozzi had taken. Mathematics is clear and superior because the nature of its subject matter is clear.

In the context of the Collegio Romano, in which Clavius was specifically in conflict with the natural philosophers, Clavius appeals to the superiority of mathematics through a comparison with the natural philosophers. To help him make this point he creates an illustrative analogy in which Aristotle is a tree trunk with the various

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[184] Lattis, *Between Copernicus and Galileo*, p. 17.

[185] Clavius, *Euclidis Elementorum*, p. a7v. It is likely that they were good friends, since they wrote letters to each other. See Sasaki, *Descartes's Mathematical Thought*, p. 51.

[186] Clavius, *Euclidis Elementorum*, p. b1v: "Quoniam disciplinae Mathematicae de rebus agunt, quae absque ulla materia sensibili considerantur."

followers of Aristotle described as tree branches coming out of the trunk. He then contrasts this with Euclid's theorems, which he states were just as certain many years ago as they were in Clavius's own time period. His point here is to contrast the obscurity of meaning that has developed in natural philosophy with the clarity in mathematics. The meaning of Aristotle has varied among his various followers, but mathematics has remained consistent. In order to legitimate his claim Clavius cites from Plato's *Philebus* that "the science superior in sincerity and truth is more worthy and excellent."[187] Clavius's point behind this analogy is clear. If the methods of mathematics were not enough to convince someone, then a comparison of it with that of natural philosophy should help strengthen the case. The nature of mathematics, both historically and methodologically, are clear. The nature of natural philosophy, while considered clear, in reality is quite obscure.

In his appeal for mathematics, it is noticeable that Clavius appealed to platonic texts. This is especially true in the section, "Various Uses of the Mathematical Disciplines." Towards the beginning of the section Clavius repeats Proclus's injunction that the mathematical disciplines enable one to arrive at the entrance of metaphysics. The clearest connection with a platonic framework follows this, in which he utilizes Plato's cave allegory from Plato's *Republic* VII. In Plato's original telling of the narrative, Plato describes individuals chained in a cave who are watching the shadows of images being projected onto a wall. The individuals, though, do not realize they are only watching shadows. They believe they are watching the actual objects. Plato utilizes

[187] Clavius, *Euclidis Elementorum*, p. b2r: "Eam scientiam esse dignitorem, praestatioremque, quae magis synceritatis, veritatisque est amans." Cf. Plato, *Philebus*, 57D.

this allegory to indicate the difference between the illusory knowledge of the sensory world, the shadows, with the true knowledge based on the ideal forms, the objects from which the shadows are formed.[188]

In the way that Clavius recasts the analogy, it is the physicists who are chained in the room, and consequently remain blind.[189] Having already presented the fact that mathematics leads to metaphysics, which he had borrowed from Proclus, Clavius's point of the narrative becomes clear. Mathematics enables one to achieve absolute knowledge, whereas physics only grants one a shadow. Such a characterization on the part of Clavius clearly connects mathematics with a platonic framework. And, if to put at ease any question about the integration of platonism with Christian theology, Clavius affirms that the Christian patriarch, Saint Augstine, was in agreement. As Clavius records, in *De Doctrina Christiana*, Augustine pointed out that math was necessary for the contemplation of divine truths.[190] Clavius then lists those who had come after Saint Augustine who had understood this truth, including in his list Saint Jerome, Gregory of Nazianus, and Saint Basil.

Based upon Clavius's Prolegomena, it is evident that he was deeply influenced by Francesco Barozzi. However, more than merely following Barozzi, Clavius appears to have in mind the specific context of the institution of the Collegio Romano.[191] And, similar to others involved in the debate over the certitude of mathematics, Clavius

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[188] See *Republic* VII 514a-517a.

[189] Ibid., p. b2v

[190] Ibid., p. b2v.

[191] This may be understood in two documents Clavius wrote in the 1580s about the institutional necessity and pedagogical benefit of mathematics. See Christoph Clavius, "A Method of Promoting Mathematical Studies in the Schools of the Society," and "On Teaching Mathematics," *Bulletin of the American Association of Jesuit Scientists* 18 (1941): p. 203-208.

appeals to a combination of Plato and Proclus to establish the legitimacy of mathematics. However, most importantly, Clavius has indicated the way in which both Proclus and Plato's ideas are connected to Saint Augustine, but also a church tradition afterwards, in order to legitimize such a connection. In so doing Clavius transformed the mathematical framework of Proclus and Plato into a Christianized form. In the process, then, Clavius transformed the meaning and significance of both Proclus and Plato into a new form. This suggests that in the discourse on mathematics Clavius felt himself free to adapt the particulars to meet his specific situation. Nevertheless, it is important to recognize that despite the changes, he still appeals to a sixteenth-century discourse on mathematics, in which mathematics is recognized as being rooted in Platonic texts, and which also made use of Proclus's prologues.[192]

MATHEMATICS AFTER CLAVIUS: JOSEPHUS BLANCANUS

Before concluding the section on the Jesuit involvement in the debate, it is worth briefly considering the way in which one of Clavius's students at the Collegio Romano, Josephus Blancanus, also known as Giuseppe Biancini, understood the debate over the certitude of mathematics. In 1615 Blancanus published *A Treatise on the Nature of Mathematics Along with a Chronology of Outstanding Mathematicians* in which he

[192] The contemporary historian Guy Claessens has written about the fact that Clavius did not properly interpret Proclus. However, the issue is not whether or not Clavius properly interpreted him, but instead the fact that Clavius did in fact appeal to him. See Guy Claessens, "Clavius, Proclus, and the Limits of Interpretation: Snapshot-Idealization Versus Projectionism," *History of Science* 47, no. 3 (2009): p. 317-336.

provides his own framework for the mathematical sciences.[193] In this text Blancanus provides his own analysis of the debate, highlighting in particular Alessandro Piccolomini's role in everything, who "attempted to rob geometers of perfect demonstrations."[194] Blancanus's text provides a very detailed recounting both of Piccolomini's opinions as well as Blancanus's own response. What emerges is an opinion quite similar to that of Clavius. Based upon the comprehensiveness with which he analyzed the debate, it also suggests that the consideration of the certitude of mathematics was something that extended beyond the 1570s, particularly within the Jesuit Order.[195]

One aspect in particular that emerges from Blancanus's recounting of the debate is his understanding of the main textual authorities involved. According to Blancanus there were three main authorities involved: Plato, Aristotle, and Proclus.[196] Throughout his *Treatise* Blancanus evidences the same reliance on Proclus as the authoritative interpreter of mathematics as has been seen in other authors. For Plato and Aristotle, though, Blancanus focuses much more clearly on each of their understandings of mathematics. With respect to Plato, Blancanus provides an extensive analysis of *Republic* Book VII. Throughout his analysis it becomes evident that the interpretation of Plato's *Republic* that Blancanus is most interested in refuting is the idea that geometers "dream" about quantity. Instead, Blancanus defends the fact that Plato said

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[193] Josephus Blancanus, *A Treatise on the Nature of Mathematics along with a Chronology of Outstanding Mathematicians*, translated by Gyula Klima, in Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, p. 178-212.

[194] Ibid., p. 184.

[195] Ibid., p. 187.

[196] Ibid., p. 187.

that geometers dream about “essence” and not about “quantity.” To help him prove his point he reconstructs what Plato was trying to accomplish in his *Republic* Book VII. According to Blancanus, Plato was establishing the proper education of the guards and governors of the Republic. To help establish the priority of education Plato integrated his understanding of the speculative sciences with the proper methods of education. Dialectics was at the top, mathematics in the middle, and opinion at the bottom.[197]

Situating each of these within Plato’s hierarchy of knowledge, Blancanus makes clear that the movement of education was toward dialectics. Moreover, as the middle discipline, mathematics enabled one to move from the physical realm, considered by physics, to the realm of dialectics. Blancanus’s point in describing this as such was to indicate that when geometers “dream” they do not dream with respect to the physical realm, but with respect to the dialectical realm. However, since he had already established that dialectics was the highest form of knowledge for Plato, and hence more real, then for a mathematician to "dream" of dialectics was to dream of reality.[198] As evidenced by his extensive explanation of Plato's classification of the sciences, Blancanus's point, then, is that Piccolomini and Pereira misinterpreted the meaning of Plato's *Republic* VII based upon misunderstanding Plato's hierarchy of the speculative sciences.

Such a distinction between an Aristotelian theory of demonstration and a Platonic framework of the speculative sciences, especially as both relate to Euclid's *Elements*, may also be seen in the way in which Blancanus concludes the book. In an appendix attached to this work, Blancanus carefully connects all of Book I of Euclid’s

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[197] Ibid., p. 197.
[198] Ibid., p. 197-198.

Elements with the specific type of Aristotelian demonstration.[199] What is most interesting is that at the end of this appendix, Blancanus mentions that others have asked that he would do the same thing for Plato's mathematical texts. However, Blancanus discovered that a Greek individual, Theon, had already completed this a long time ago. Having refused the request, Blancanus adds that, "it is the task of contemporary Platonists, lest what is done should be done again, finally to prepare a Latin edition."[200] Based on the way in which Blancanus drew an Aristotelian understanding of Euclid into relationship with a Platonic view of Euclid, it becomes evident that a significant way in which Blancanus interpreted the debate over the certitude of mathematics was with regard to a conflict between a Platonic and an Aristotelian understanding of mathematics. Though his perspective was forty years after that of Clavius and Pereira, it is still an important clue to the way the textual divide between Plato and Aristotle was influencing all ontological discussions of mathematics.

[199] Ibid., p. 208-212.
[200] Ibid., p. 212.

CONCLUSION: PLATO AND ARISTOTLE AMIDST THE *CERTITUDINE MATHEMATICARUM* DEBATE

None of the authors analyzed denied the certainty of mathematical demonstration. Moreover, they all believed that the clarity of the methods of mathematical demonstration were its greatest strength. However, what was disagreed upon was the exact nature of mathematics. The significance of this was essential for understanding how mathematics could be applied to the actual world. What exactly was the relationship of arithmetic and geometry with sensible matter? Did mathematical demonstrations describe essential aspects of objects? Or were they merely accidental? In a context in which Aristotelian demonstration was largely regarded as the most authoritative form of reasoning, such questions were of deep significance for understanding the nature of mathematics.

When Alessandro Piccolomini first stated that mathematics was not a *demonstratio potissima*, his claim was deeply rooted in the fact that he was a logician. His prevailing assumption throughout the entirety of the *Commentarium* was that the *potissima* presented the essential form of knowledge. However, since mathematics itself, according to him, were essentially abstractions of the imagination, then, mathematics could not fit the framework of the *potissima*. A significant part of his argument was the way in which he utilized Aristotle, Plato, and Proclus to demonstrate his point. Aristotle provided for him the nature of the *potissima* and the rules under which it operated. And Proclus, largely understood as a follower of Plato, provided the nature of mathematics. Beneath the surface of his criticism was the fact that these sources could not be reconciled with each other.

Francesco Barozzi's response demonstrated this. As indicated, a significant part of Barozzi's response to Piccolomini was that the methods of mathematics demonstrate their certainty. Equally significant was the fact that Barozzi emphasized that the debate over the certitude of mathematics hinged on the way in which Plato, Aristotle, and Proclus were understood. As he indicates in the *Opusculum*, his translation of Proclus's *Commentary* was intended to help demonstrate the relationship between Plato and Aristotle to mathematics. For Barozzi, Proclus's *Commentary* served as a unifier between the two. Moreover, quite clearly in his *Opusculum*, it was Barozzi's belief that the only difference between a Platonic and an Aristotelian understanding of the sciences was one of perspective.

Such a divide between the abstraction of mathematics and the clarity of its methods carried on into the Jesuit Order. Benito Pereira borrowed the same basic framework that Piccolomini had presented. However, more so in Pereira than in Piccolomini, it is evident that Pereira considered mathematics to be essentially Platonic in its nature. This was seen in the way that he included the Platonic concept of dialectics in the context of his refutation of mathematics, but also in the way in which he guarded against the theory of reminiscence. For the Jesuit, the way in which he understood mathematics was connected to wider philosophical issues. Christoph Clavius, on the other hand, was most interested in understanding mathematics due to its widespread utility. Quite clearly in his texts as well, it is evident that he understood mathematics as essentially developing from Platonic ideas. And quite provocatively, the clarity of mathematics was best understood in contrast to the obscurity of physics.

What has emerged through this study is the way in which the conflict that ensued within this debate occurred as a result of a divide between an Aristotelian theory of demonstration and a Platonic theory of mathematics. At the time in which the debate was occurring it appears that the prevailing theory of mathematics was one rooted in Plato, and that part of what was involved was how to merge an Aristotelian theory of demonstration and of reality with a Platonic foundation of mathematics. Such an understanding certainly fits with Blancanus's own understanding. After summarizing the debate, he provided an analysis of Euclid's Book I in relation to Aristotle and indicated that others desired the same for Plato.

At the beginning of this essay it was pointed out that the prevailing analysis for the debate over the certitude of mathematics was that it shaped the development of Jesuit mathematics. If the thesis of this essay is correct, then Jesuit mathematics ought to be understood within a context of mathematical development in which a significant issue was the reconciliation of Plato and Aristotle. Mathematics was predominately understood as being founded on the texts of Plato, largely *Republc* VII. As a consequence an important aspect of this debate was how to reconcile the nature of mathematics as established in these texts with an Aristotelian understanding of demonstration. Such reconciliation naturally invoked the classification of the sciences, bringing the systems rooted in Aristotle in comparison with Plato. It was within this context that the nature of mathematics, and its ontology, at least among the University trained, was being analyzed.

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